

## OFDMA-based multicast with multiple base stations

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**Abstract:** We consider an orthogonal frequency division multiple access (OFDMA)-based multicast system where multiple base stations transmit a multicast session to a multicast group. The goal is to maximize the multicast rate (i.e. the minimum achievable user rate) subject to a total power constraint. We assume the use of an erasure code (e.g., a Reed–Solomon code) or rateless code (e.g., Luby transform code). This facilitates each user to accumulate rates from their best subchannels, so that the achievable multicast rate is not limited to the worst user. The resource allocation problem involves determining the transmitting base station at each OFDMA subchannel, the number of bits to be transmitted at each subchannel, and the set of nodes to decode the bits at each subchannel. We formulate the problem as a mixed binary integer linear programming and find the optimal solution as a benchmark. We also propose a greedy subchannel and bit allocation algorithm that is close to optimal.

**Key words:** Multicast, orthogonal frequency division multiplexing, erasure codes, rateless codes, multiple base stations

### 1. Introduction

Orthogonal frequency division multiplexing (OFDM) is one of the promising techniques to combat frequency-selective fading in broadband wireless systems. With this technique, the information is sent over narrowband transmission channels named subchannels. In a multiple-user system, where each user experiences different frequency selective fading, OFDM can be used as a multiple access technique (OFDMA) by judiciously allocating subchannels to users.

Wireless multicast/broadcast services such as IP radio broadcast and mobile TV is a growing application in broadband wireless access systems [1]. As individual feedback-based systems such as ARQ/HARQ cannot be supported in multicast systems due to the large amount of feedback required, robust modulation and coding schemes are used, which limits the throughput. Therefore, multicast transmission in OFDM-based systems receives much attention. Traditionally in multicast resource allocation, the base station looks at the user in the multicast group with the worst channel gain [2]. In an OFDM-based system, the multicast throughput of each subchannel is determined by the user with the worst channel gain in the corresponding channel. In [2], total throughput is maximized in the presence of multiple multicast groups. Some lower complexity resource allocation schemes are also proposed in this paper. The work in [3,4] improves the total throughput by using the benefits of OFDM along with multiple description coding [5]. In that scheme, the information (preferably a video, speech, or image) is coded in layers. Users with better channel conditions receive the same broadcast, but with better quality. However, the work in [4] did not consider fairness and the users with bad channel

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conditions had fewer benefits. The received rate of the user with the worst channel significantly decreases as the number of users in the group increases.

The work in [6] significantly improves the throughput of the worst user by using a technique based on Reed–Solomon coding. Available information is coded using an erasure code such as Reed–Solomon codes, and the receiver is able to decode the information as long as it can successfully receive a sufficient amount of coded data. This way, each user can receive information using its best subchannels and the subchannel capacities are no longer limited to the worst-user channel conditions. In [6], the case of multiple multicast groups was also studied. The goal in that work was to maximize the multicast rate (in the case of a single group) and maximize the minimum multicast rate (in the multiple session case). In [7] we based our work on [6] and proposed an optimal solution to the problem using mixed binary integer and linear programming. We also proposed a suboptimal algorithm, which has better performance than that of [6]. The performance and complexity improvement becomes more significant as the number of users and groups increase. The work in [8] studies multihop multicast, where the nodes that are able to decode the information can serve as relays.

This work is also related to the literature on rateless codes and mutual information accumulation. Rateless (fountain) codes are application layer erasure codes [9,10], where the source has a number of data packets and sends each time a randomly selected and XOR'ed combination of those packets. Here there is no need to receive each and every transmitted packet. The receiver only needs to accumulate a number of coded packets in order to decode the original data. Moreover, in the case of multiple sources, a receiver is able to accumulate packets from multiple sources. In a multihop scenario a node can accumulate packets that are transmitted by the nodes in previous hops of the routing path. If we idealize this situation we can assume that mutual information is accumulated instead of packets. There are some recent works such as [11–14] that studied optimal routing in wireless networks in the presence of mutual information accumulation.

Contrary to the previous work, in this work we assume the use of multiple base stations, where the base station transmissions are coordinated in order to maximize the multicast rate, subject to a total power constraint. Here the problem involves determining the base station to transmit at each subchannel, the number of bits to transmit at each subchannel, and the set of users that decode each subchannel. A base station that is good for a user can be far from others, which increases the importance of the base station assignment problem.

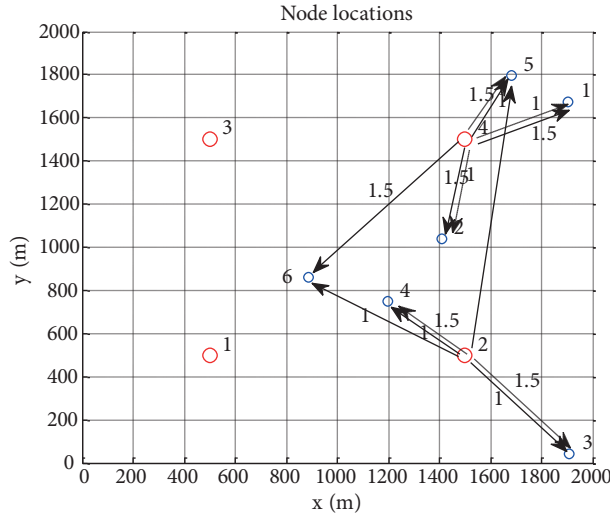
The method proposed in this work can also be considered as a type of coordinated multipoint transmission [15], where multiple base stations (BSs) are distributed over the cellular area and coordinate their transmission to improve the spectral efficiency or coverage area. This coordinated transmission can also be achieved with distributed antenna systems. In a multiple-BS scenario the resource allocation decision may be given by a base station coordinator, and for a distributed antenna system the same decision can be given by the base station.

## 2. System model and problem formulation

A set of  $S$  base stations transmit a multicast session to a group of  $K$  users (Figure 1). The total bandwidth  $W$  is divided into  $N$  subchannels of bandwidth  $W_{sub} = W/N$ . Each subchannel can be used by only one base station; therefore, there is no interference. The case where multiple BSs can transmit on the same subchannel may improve the energy efficiency and is a topic for future research.

The channel gain between the BS  $s$  and user  $k$  in subcarrier  $n$  is denoted by  $h_{n,s,k}$  and is assumed to be known by the controller. This channel gain includes distance-based path loss, shadowing, and Rayleigh fading. Each subchannel experiences flat fading, which also experiences an additive white Gaussian noise of power spectral density  $N_0W_{sub}$ . In order to be able to decode the multicast message, each user has to accumulate a

rate of  $R_0$  bps. A type of erasure coding (e.g., Reed–Solomon code or fountain code) is used so that if a user is able to decode more than a specific number of bits, it is able to decode the multicast message. Each user can accumulate the required  $R_0$  bps rate from different subchannels. Resource allocation involves: 1) assigning a BS to each subchannel, 2) deciding the set of users in the group that will decode the subchannel, and 3) the number of bits to transmit at each subchannel. The transmitted rate from a subchannel is assumed to be chosen from a discrete set  $C$ , where  $C = \{C^1 = 0, C^2, \dots, C^M\}$ , where  $CM$ , is the maximum number of bits that can be transmitted in a subchannel.



**Figure 1.** BS 2 transmits in subchannel 1 with rate 1.5 bps/Hz to nodes 3 and 4. BS 2 also transmits in subchannel 4 with rate 1 bps/Hz to nodes 3, 4, 5, and 6. BS 4 transmits in subchannel 2 with rate 1.5 bps/Hz to nodes 1, 2, 5, and 6. BS 4 also transmits in subchannel 3 with rate 1 bps/Hz to nodes 1, 2, and 5. Subchannel 5 is idle.

Depending on the bit error ratio (BER) requirement, each of these rates corresponds to a minimum SNR value that is required to receive that rate, which is denoted by  $f_m, m = 1, \dots, M$ , where  $f_1 = 0$ . We define the indicator  $x_{n,k}^{s,m}$ , which is equal to one, if BS  $s$  transmits in subchannel  $n$ , with rate  $cm$ , which is received by the best  $k$  users. We also define the rate matrix  $R_{n,k}^{s,m}(i)$  for each user  $i$ .  $R_{n,k}^{s,m}(i)$  takes value  $c_m$  if BS  $s$  transmits from subchannel  $n$  with rate  $c_m$  and user  $i$  is among the best  $k$  users from the BS  $s$ , on subchannel  $n$ . We also define a power matrix

$$P_{n,k}^{s,m} = \max_{i \in \pi_k^{n,s}} \left\{ \frac{f_m N_0 W_{sub}}{h_{n,s,i}} \right\}, \forall n \in N, s \in S, k \in K, m \in M \tag{1}$$

where  $\pi_k^{n,s}$  is the set of best  $k$  nodes from BS  $s$  on subchannel  $n$ . This means the power required for the best  $k$ th node from BS  $s$ , in subchannel  $k$ , for rate  $c_m$ . The optimal resource allocation problem is formulated as follows.

$$\max_x R_0 \tag{2}$$

s. t.

$$P_T \geq \sum_{n \in N} \sum_{s \in S} \sum_{k \in K} \sum_{m \in M} x_{n,k}^{s,m} P_{n,k}^{s,m} \tag{3}$$

$$R_0 \leq \sum_{n \in N} \sum_{s \in S} \sum_{k \in K} \sum_{m \in M} x_{n,k}^{s,m} R_{n,k}^{s,m}(i), \forall i \in K \tag{4}$$

$$1 \geq \sum_{s \in S} \sum_{k \in K} \sum_{m \in M} x_{n,k}^{s,m}, \forall n \in N \quad (5)$$

$$x_{n,k}^{s,m} \in \{0, 1\}, \forall n \in N s \in S k \in K m \in M \quad (6)$$

The objective function of Eq. (2) is for the maximization of the minimum achievable user rate in the multicast group. The constraint of Eq. (3) is the total power constraint. The constraint of Eq. (4) assures that each user exceeds  $R_0$ . The constraint of Eq. (5) is the requirement that a single BS, a single rate, and a single number of best  $k$  users should be determined for each subchannel. Lastly, the constraint of Eq. (6) is the binary integer constraint for the decision variable  $x_{n,k}^{s,m}$ . This problem can be solved by mixed binary integer-linear programming solvers, such as CPLEX. GAMS is an optimization software that includes CPLEX and several other solvers that can solve various types of optimization problems. For this purpose, the decision and power matrices are converted into vectors. The rate matrix for each user is also converted into a vector. The constraints of Eqs. (4) and (5) can be written as matrix inequalities. The optimization is performed using GAMS.

For the multiple-BS scenario, we assume that the data to be transmitted are available at each BS, and the resource allocation decisions are made by a base station controller in a centralized manner.

### 3. Proposed algorithm

Let us assume that the total power  $P_T$  is divided equally over the subchannel ( $P_T/N$ ). Let  $b_{k,n,s}$  be the index of number of bits that can be transmitted from station  $s$  to node  $k$  using subchannel  $n$  with  $P_T/N$  transmission power ( $C_{b_{k,n,s}}$  is the received rate in bps/Hz). Let  $s_n$  be the base station that transmits in subchannel  $n$ ,  $c_n$  be the rate index that is used in subchannel  $n$  ( $C_{c_n}$  is the transmitted bps/Hz), and  $p_n$  be the power expenditure in subchannel  $n$ . The proposed algorithm consists of three stages.

The first stage described in Algorithm 1 divides the available power equally among the subchannels. Instead of maximizing the user rate, the algorithm minimizes  $\sum_k \left(\frac{1}{R_k + \epsilon}\right)^\gamma$  as the utility function.

Here  $\gamma$  is a large number; therefore, the utility function is dominated by the minimum-rate user. This type of utility function is preferred because the users start the algorithm with zero rates, and if we used the maximum rate the first subchannel and bit allocation may still result in zero minimum-rate; therefore, the algorithm would not be able to proceed. On the other hand, the proposed utility function can be improved with each new allocation decision, and for large  $\gamma$  it is equivalent to maximizing the minimum user rate. The parameter  $\epsilon$  is a small number and avoids division by zero.

Lines 4–25 are the main loop. In lines 5–24 the algorithm checks each subchannel considering the allocations for other subchannels as fixed. For each subchannel the algorithm checks possible rate allocations (line 7) and base stations (line 8) for that subchannel. The algorithm finds the resulting multicast rate (line 12) corresponding to each bit and base station allocation and finds the allocation that minimizes the utility function (line 19). If no improvement can be obtained (that is, if the new multicast rate after checking all subchannels is the same), then the algorithm is finished. At the end (line 26), the algorithm finds the minimum required power in order to transmit the decided number of bits to the given set of users from the given base stations. As all the subchannels, bit rates, and stations are checked at each iteration, the complexity of each iteration is  $O(NMS)$ . The complexity of Stage 1 becomes  $O(NMSI_1)$ , where  $I_1$  is the number of iterations (i.e. number of turns of the WHILE loop).

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**Algorithm 1.** Proposed Algorithm : Stage 1.

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- 1: Initialize  $b_{k,n,s} = 1, \forall n, k, s, c_n = 1, \forall n, s_n = 0, \forall n, p_n = \frac{P_T}{N}, \forall n$
  - 2: Calculate  $b_{k,n,s} = \underset{m=1 \dots M}{\operatorname{argmax}} \left\{ \frac{P_T}{N} \frac{h_{k,n,s}}{N_0 W_{sub}} \geq f_m \right\}, \forall k, n, s$
  - 3:  $R_k = 0, \forall k \in K, U_{min} = \sum_k (1/(R_k + \epsilon))^Y$
  - 4: **while** there is no improvement **do**
  - 5:   **for**  $n = 1:N$  **do**
  - 6:     Initialize  $m^* = 0, s^* = 0$
  - 7:     **for**  $m = 1:M$  **do**
  - 8:       **for**  $s = 1:S$  **do**
  - 9:         Set  $R'_k = R_k$
  - 10:         Set  $w'_{k,n} = w_{k,n}, \forall k, n$
  - 11:         Set  $w'_{k,n} = m$  if  $b_{k,n,s} \geq m, w'_{k,n} = 1$  else,  $\forall k$
  - 12:         Calculate  $R'_k = \sum_{n=1}^N C_{w_{k,n}}, \forall k$
  - 13:         **if**  $\sum_k \left( \frac{1}{R'_k + \epsilon} \right)^Y < U_{min}$  **then**
  - 14:             Set  $U_{min} = \sum_k \left( \frac{1}{R'_k + \epsilon} \right)^Y$
  - 15:             Set  $m^* = m, s^* = s$
  - 16:         **end if**
  - 17:         **end for**
  - 18:         **end for**
  - 19:         **if**  $m^* > 0$  **then**
  - 20:             Set  $s_n = s^*, c_n = m^*$
  - 21:             Set  $w'_{k,n} = m^*$  if  $b_{k,n,s^*} \geq m^*, w'_{k,n} = 1$  else,  $\forall k$
  - 22:         **else**
  - 23:         **end if**
  - 24:         **end for**
  - 25:     **end while**
  - 26: Calculate subchannel powers  $p_n = \underset{k \text{ s.t. } w_{k,n}, s_n > 1}{\operatorname{max}} \left\{ \frac{f_{c_n} N_0 W_{sub}}{h_{k,n}, s_n} \right\}, \forall n$
- 

Stage 2 of the proposed algorithm in Algorithm 2 finds the unnecessary power allocations and decreases the total power allocation (without decreasing the multicast rate), so that Stage 3 can be better utilized to improve the multicast rate.

**Algorithm 2.** Proposed Algorithm: Stage 2.

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1: while there is power decrease do
2:   Set  $\Delta p_n = 0, \forall n$ 
3:   for  $n = 1:N$  do
4:     if  $c_n > 1$  then
5:       Set  $w'_{k,n} = w_{k,n}, \forall k, n$ 
6:       Set  $w'_{k,n} = \max(1, w_{k,n} - 1), \forall k$ 
7:       if  $\min_k \left\{ \sum_n^N C_{w'_{k,n}} \right\} \geq \min_k R_k$  then
8:          $\Delta p_n = p_n - \max_{k \text{ s.t. } w'_{k,n}, s_n > 1} \left\{ \frac{f_{c_n} N_0 W_{sub}}{h_{k,n}, s_n} \right\}$ 
9:       end if
10:    end if
11:  end for
12:  if  $\max_n(\Delta p_n) > 0$  then
13:     $n^* = \arg \max_n(\Delta p_n)$ 
14:    Set  $w_{k,n^*} = \max(1, w_{k,n^*} - 1), \forall k$ 
15:    Update subchannel power  $p_{n^*} = \max_{k \text{ s.t. } w_{k,n^*}, s_{n^*} > 1} \left\{ \frac{f_{c_{n^*}} N_0 W_{sub}}{h_{k,n^*}, s_{n^*}} \right\}$ 
16:  else
17:    Finish
18:  end if
19: end while

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Stage 2 consists of a main loop (lines 1–19). In the main loop the algorithm checks each subchannel (lines 3–11) and sees if the rate allocation  $c_n$  can be decreased by one level without decreasing the minimum user rate (line 7). If yes, the algorithm calculates the power savings  $\Delta p_n$  by that change (line 8). The algorithm then finds the subchannel that facilitates the minimum power decrease and decreases its number of bits by one level (lines 13–15). If no decrease can be obtained, the stage finishes (line 17). Stage 2 looks at the  $N$  subchannels at each iteration of the WHILE loop. Therefore, the complexity of Stage 2 becomes  $O(NI_2)$ , where  $I_2$  is the number of iterations of the WHILE loop.

As the result of Stages 1 and 2, there may be a significant amount of residual power that can be allocated in order to improve the minimum user rate.

Stage 3 of the algorithm calculates the residual power and allocates to the subchannels in order to maximally increase the multicast rate of the group. It is a modified version of the Levin–Campello algorithm [16] and was originally proposed in [6] in order to use the residual power, load more bits, and improve the multicast rate.

**Algorithm 3.** Proposed Algorithm: Stage 3.

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1: while there is improvement do
2:   Calculate:  $P_{res} = P_T - \sum_n p_n$ 
3:   Calculate:  $h_n^{min} = \min_{k:w_{k,n} > 1} \{h_{k,n,r_n}\}, \forall n$ 
4:   Calculate:  $\Delta p_n = \frac{N_0 W_{sub}(f_{c_{n+1}} - f_{c_n})}{h_n^{min}}$  if  $c_n < \mathbf{M}$ ,  $\Delta p_n = \infty$  else
5:   Find  $k^* = \operatorname{argmin}_k R_k$ 
6:   Find  $n^* = \operatorname{argmin}_{n:w_{k^*,n} > 1} \{\Delta p_n\}$ 
7:   if  $\Delta p_{n^*} < P_{res}$  then
8:     Update  $c_{n^*} = c_{n^*} + 1$ ,  $p_{n^*} = p_{n^*} + \Delta p_{n^*}$ 
9:     Update  $w_{k,n^*} = w_{k,n^*} + 1$  if  $w_{k,n^*} = c_{n^*}$ ,  $w_{k,n^*} = 1$  else  $\forall k$ 
10:    Update  $R_k = \sum_n C_{w_{k,n}}, \forall k$ 
11:   else
12:     finish
13:   end if
14: end while

```

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Stage 3 also consists of a main loop (lines 1–14). It first calculates the residual power in line 2. It then calculates for all subchannels the power required to increase the number of bits one step further (lines 3 and 4). Then the minimum-rate user  $k^*$  is found (Line 5). If there is more than one minimum-rate user, then one of them can be chosen arbitrarily. Among the subchannels allocated to  $k^*$ , the one  $n^*$  that requires the least amount of additional power is found (Line 6). The bit index  $c_n^*$  allocated to subchannel  $n^*$  is increased one step further. Residual power is updated. The algorithm continues until the residual power is not enough to increase the minimum user rate (line 12). As for the complexity, in real implementations, lines 3 and 5 can be computed before the WHILE loop and can be updated after line 10, which helps to decrease the complexity. The complexity is  $O(NKI_3)$ , where  $I_3$  is the number of iterations of the WHILE loop.

#### 4. Benchmark algorithms

To the best of our knowledge, this is the first study that focuses on OFDM-based multicast with multiple base stations and erasure/fountain coding. Therefore, we decided to evaluate our algorithm in comparison to a less intelligent algorithm that consists of a simple base station and bit allocation.

The benchmark in Algorithm 4 consists of two stages, base station allocation and bit allocation. The first stage is very simple and it allocates subchannels to base stations in a round-robin fashion (for example, subchannels from 1 to 25 are allocated to 4 base stations,  $s_1 = 1, s_2 = 2, s_3 = 3, s_4 = 4, s_5 = 1, s_6 = 2, s_7 = 3, s_8 = 4, \dots, s_{24} = 4, s_{25} = 1$ ). According to this algorithm, equal numbers of bits are transmitted at each subchannel. This stage checks each bit index from  $C_1$  to  $C_M$ , calculates the achievable minimum user rate, and selects the maximizing one. The complexity of the benchmark algorithm is  $O(KNM)$ , because the

algorithm assigns BSs to subchannels with no complexity. Then it checks all bit rates one by one and calculates the achievable rates of all users in all subchannels.

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**Algorithm 4.** Benchmark Algorithm.

---

1: Set  $b_{k,n,s} = 1, \forall n, k, s, c_n = 1, \forall n, s_n = 0, \forall n, p_n = \frac{P_T}{N}, \forall n, \text{counter} = 1$

2: BASE STATION ALLOCATION

3: **for**  $n = 1 : N$  **do**

4:      $s_n = \text{counter}$

5:      $\text{counter} = \text{counter} + 1$

6:     **if**  $\text{counter} = S$  **then**

7:          $\text{counter} = 1$

8:     **end if**

9:     **end for**

10: BIT ALLOCATION:

11: Calculate  $b_{k,n,s_n} = \underset{m=1..M}{\text{argmax}} \left\{ \frac{P_T}{N} \frac{h_{k,n,s_n}}{N_0 W_{sub}} \geq f_m \right\}, \forall k, n$

12: Initialize  $R_{max} = 0, w_{k,n} = 1, \forall k, n$

13: **for**  $m = 1 : M$  **do**

14:     Calculate  $R'_k = R_k$

15:     Set  $w'_{k,n} = m$  if  $b_{k,n,s_n} \geq m, w'_{k,n} = 1$  else,  $\forall k, n$

16:     Calculate  $R'_k = \sum_{n=1}^N C_{w'_{k,n}}, \forall k$

17:     **if**  $\min_k \{R'_k\} < R_{max}$  **then**

18:         Set  $R_{max} = \min_k \{R'_k\}$

19:         Set  $w_{k,n} = w'_{k,n}, \forall k, n$

20:         Calculate  $R_k = \sum_i C_{w_{k,n}}, \forall k$

21:     **end if**

22: **end for**

23: Calculate subchannel powers  $p_n = \max_k \text{st. } w_{k,n,s_n} > 1 \left\{ \frac{f_{c_n} N_0 W_{sub}}{h_{k,n,s_n}} \right\}, \forall n$

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We also consider a “decentralized” algorithm, where each user is only served by one BS. Each user is connected to the BS that maximizes the average channel gain. The subchannels and total power are shared equally among the BSs. The rest of the algorithm is almost identical to the proposed Algorithm 1, but it is applied to each BS separately. The advantage of this algorithm is that each BS can perform its resource allocation simultaneously, in a decentralized manner. Since the number of subchannels and users per each cell is  $S$  times less on the average, the complexities of Stages 1, 2, and 3 become  $O(NMI_1/S)$ ,  $O(NI_2/S)$ , and  $O(NKI_3/S^2)$ , respectively.



## 5. Numerical results

The solutions that we compare in this section are: 1) Optimal Solution, 2) Proposed Algorithm (Stage 1), 3) Proposed Algorithm (Stage 1+3), 4) Proposed Algorithm (Stage 1+2+3), 5) Benchmark Algorithm, and 6) Decentralized Algorithm. We generate 1000 different network instances, which correspond to different node locations, and channel gains. We computed performances of each solution corresponding to all 1000 cases. As a result we obtained five performance (multicast rate) vectors, with size 1000 each. As expected, the vector corresponding to the optimal solution is the greatest for each case. We computed the relative performance by dividing the performance vector of each solution into the optimal performance vector, element by element. We plotted the empirical cumulative distribution function of each resulting vector. This gives us the statistical information of closeness of the performance of each solution to that of the optimal.

We performed all of the simulations in a square cellular area of  $2000 \times 2000$  meters, with  $S = 4$  base stations. The base stations are always located as in Figure 1 (i.e. they are fixed). We made three separate simulations for the number of nodes equal to  $K = 20, 30, 40$ . For each set of simulations we performed several trials, which correspond to random node locations and channel conditions. We consider an OFDM-based system with  $N = 100$  subchannels, with a bandwidth of  $W_{sub} = 200$  kHz each [17]. Transmissions are subject to an additive white Gaussian noise of power spectral density  $N_0 = -174$  dBm W/Hz. Path loss is modeled as  $31.5 + 35 \times \log_{10}d$ , where  $d$  is the distance in meters. This is a standard path loss model for suburban area and nonline-of-sight transmission [18]. We assume shadow fading with a standard deviation of 8 dB. That is, the shadow fading (in dB) has a probability density function  $f_S(s) = \frac{1}{8\sqrt{2\pi}} \exp\left(-\frac{s^2}{2 \times 8^2}\right)$ . We assume a shadowing decorrelation distance of  $d_0 = 100$  m. Correlation coefficient between two locations is  $\exp(-d/d_0)$ , where  $d$  is the distance between the two locations [19]. Uncorrelated shadow fading values are generated according to  $f_S(s)$ , and then the correlated values are generated using Cholesky decomposition. Multipath fading is independent for each subchannel, user, and base station. The modulation and coding techniques used in the simulations are summarized in Table 1. For example, “QPSK 1/2 2x” means that QPSK modulation is used with 1/2-rate convolutional coding and 2 times repetition coding [20].

**Table 1.** Modulation-coding techniques, spectral efficiencies, and SNR thresholds.

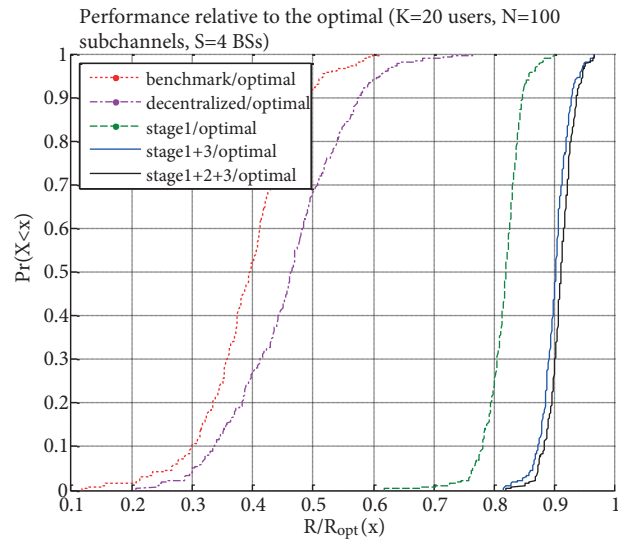
Mod/Cod	Bps/Hz	SNR threshold (dB)
QPSK 1/2 2x	1/2	2
QPSK 1/2 1x	1	5
QPSK 3/4 1x	3/2	6
16QAM 1/2 1x	2	10.5
16QAM 3/4 1x	3	14
64QAM 2/3 1x	4	18

Figure 2 shows the cumulative distribution of the algorithm performances with respect to the optimal solution. For this purpose 1000 trials are performed for each algorithm. A trial is performed using same scenario for all algorithms, for the sake of fair comparison. As a result, a performance vector of  $1000 \times 1$  is obtained for each algorithm. The relative performance vector is obtained by element-wise division of the vector’s corresponding algorithm to that of the optimal. Results in Figures 2, 3, and 4 are the cumulative empirical distribution of these vectors. The simulations are carried out for 20 users, 100 subchannels, 4 base stations, and 40 W of total power. The results show that the proposed algorithm provides significant improvement

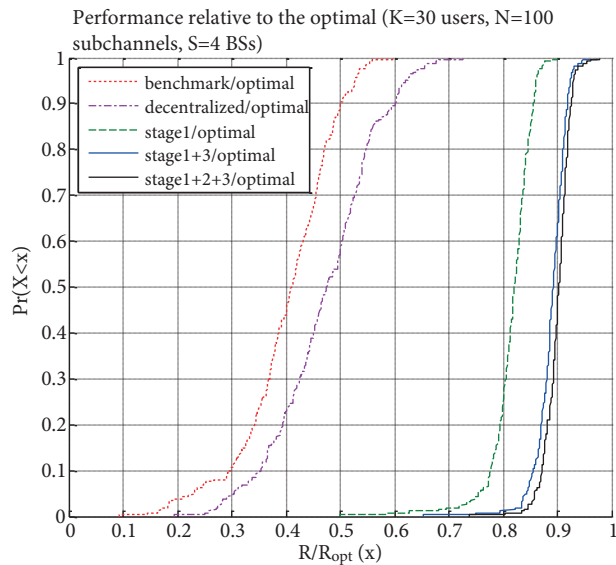
with respect to the benchmark algorithm. Even if only the first stage of the proposed solution is employed, the performance stays within 20% of the optimal most of the time. If the other stages are employed, the performance approaches to within 10% of the optimal. Most of the time, the multicast rate of the benchmark algorithm is less than half of the optimal performance. The decentralized algorithm is better than the benchmark, yet achieves only half of the optimal multicast rate.

Figures 3 and 4 show the relative performances for 30 and 40 users, respectively. The general characteristics are similar. The benchmark and decentralized solutions stay in a similar position with respect to the optimal. The relative performance of the proposed solution drops slightly; however, it still stays within 12%–13% of the optimal most of the time.

Table 2 shows the real average multicast rate performances of the solutions. We see that a multicast rate of 20–25 Mbps can be obtained by the optimal solution. Performance of the proposed algorithm with at least Stage 1 and 3 is quite close to the optimal performance. On the other hand the multicast rate of the benchmark solution can only achieve 40% of the optimal for 40 users.



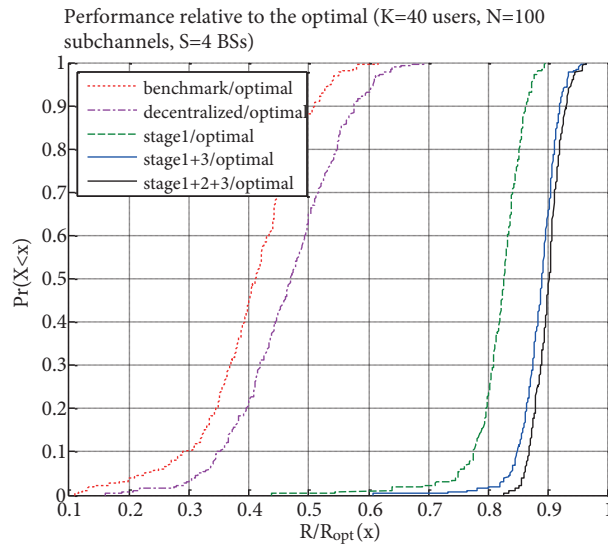
**Figure 2.** Cumulative distribution of the multicast rate performance with respect to the optimal (20 users, 100 OFDM subchannels, 4 base stations, and 40 W total power).



**Figure 3.** Cumulative distribution of the multicast rate performance with respect to the optimal (30 users, 100 OFDM subchannels, 4 base stations, and 40 W total power).

**Table 2.** Average multicast rates (Mbps):  $N = 100$  subchannels,  $S = 4$  stations,  $P = 40$  W.

Solution	20 users	30 users	40 users
Benchmark	9.64	8.58	8.04
Decentralized	11.23	10.06	9.20
Proposed: Stage 1	19.81	17.25	16.09
Proposed: Stage 1+3	21.85	18.78	17.38
Proposed: Stage 1+2+3	22.07	19.00	17.65
Optimal	24.24	21.09	19.61



**Figure 4.** Cumulative distribution of the multicast rate performance with respect to the optimal (40 users, 100 OFDM subchannels, 4 base stations, and 40 W total power).

## 6. Conclusions

We considered the problem of OFDM-based multicast transmission with multiple base stations. The optimal resource allocation problem involves determining the optimal base station to transmit at each subchannel, along with transmission rate and the set of decoding nodes at each subchannel. We formulated the resource allocation problem as mixed binary integer-linear programming, which can be solved using off-the-shelf optimization tools. We also proposed a greedy suboptimal algorithm that increases the total accumulated rates of the users at each step, in a way that requires minimal additional power. Numerical evaluations show that the proposed greedy algorithm provides significant improvement with respect to a less intelligent algorithm.

In this work we assumed that each subchannel is used by only one base station. In reality, the base stations further away may be able to transmit simultaneously, so that each user can receive from the base station closest to it. This results in interference and makes the optimization problem nonlinear. Although the problem becomes harder, with clever resource allocation algorithms, simultaneous transmission of base stations may result in better multicast throughput.

In this work we also assumed an aggregate power constraint for the base station. This can be assumed in a distributed antenna system. However, for the case of separate base stations, power sources (and hence power constraints) must be separate. This constraint can be easily introduced into the optimization code. However, suboptimal practical algorithms become harder to design.

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