ESSAYS ON REGULATION IN MACROECONOMICS

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#### ABSTRACT

#### ESSAYS ON REGULATION IN MACROECONOMICS

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This thesis is composed of three papers that explore the macroeconomic implications of regulating externalities. The first two papers aim to contribute to the literature on the interaction between macroeconomic and environmental policies, focusing on alternative policies to control pollution. In the third paper, we extend the discussion on price versus quantity controls to the realm of capital inflow control measures.

In the first paper, we explore the interaction of monetary policy and a regulatory policy for controlling pollution within an economy populated with financially constrained producers exhibiting heterogeneity in production technology and pollution rates. Environment related components of the model include pollution externality, an abatement technology and environmental policy in the form of tax on pollutants. Our analysis is organized around two main topics: assessing the effect of monetary policy on social welfare in the presence of environmental concerns and investigating how the existence of pollution-type externality and environmental regulation influences optimal monetary policy. Our findings suggest that in the presence of heterogeneity, due to its distributional impact, monetary policy can play a role in enhancing social welfare and complementing regulatory efforts to mitigate pollution.

The primary question of interest in the second paper revolves around how the economy featuring a partial cash-in-advance constraint in the labor market responds to productivity shocks under different regulatory policies and how the extent of nominal rigidity affects this response. We employ a stochastic general equilibrium model featuring nominal rigidity in the form of partial cash-in-advance constraint in the labor market, pollution associated with production activity, and an abatement effort. Environmental policies include a price policy (tax) and a quantity policy in the form of cap-and-trade system. Our key findings are twofold. First, volatility in macro variables is higher under price regulation compared to quantity regulation. Under quantity regulation, the cost of controlling pollution is positively associated with output. This channel reduces the response of labor and all other variables to productivity shocks. Second, we demonstrate that as the degree of nominal rigidity increases, volatility increases under both regulations but relatively more so under price regulation. In the third paper, we explore the impact of a capital inflow shock under different control measures. We employ a small open economy model incorporating learningby-doing externality in the tradable sector. Under the competitive equilibrium, consumption and external borrowing in the model exceed the socially optimal amount. Under the price regulation, the regulator's role is to set a tax on external borrowing. With quantity regulation, the regulator establishes a cap on aggregate external borrowing and issues borrowing permits, which households demand in the spot market. Under both regulations, there is information asymmetry as the regulator sets the policy before observing the interest rate. Agents, however, make decisions after observing the shock.

The third paper aims to contribute in two ways: First, drawing insights from the literature on price versus quantity controls, we compare welfare implications of price (tax) and quantity-type regulation (cap-and-trade) for capital inflows. Second, we point out the concept of a market-based regulatory framework for capital inflows.

We demonstrate that there is less volatility under quantity regulation and, in terms of utility, quantity policy outperforms price policy in the short-run. This superiority arises from the shape of the social welfare function and ex-post variation in external debt under price regulation. Furthermore, the ranking of policies is influenced by the initial productivity level. Regarding social welfare, quantity type control performs better than price control when the initial productivity level is low. Moreover, the relative advantage of price over quantity policy declines with an increase in the pace of technology growth.

Keywords: Price vs. Quantity Regulation, Pollution, Capital Inflow Control

## ÖΖ

## MAKROEKONOMİ ALANINDA KONTROL DÜZENLEMELERİ ÜZERİNE MAKALELER

## GÜRCİHAN, Hatice Burcu Doktora, İktisat Tez Danışmanı: Prof. Dr. İsmail SAĞLAM

Bu tez, dışsallıkları düzenlemenin makro ekonomik sonuçlarını araştıran üç makaleden oluşmaktadır. İlk iki makale, makroekonomik politikalar ile çevre politikaları arasındaki etkileşimi inceleyen yazına katkıda bulunmayı amaçlamaktadır. Üçüncü makale, dışşallıkları düzenlemede öne çıkan fiyat ve miktar kontrolleri kıyaslamasını sermaye girişi kontrol önlemleri alanına uygulamaktadır.

İlk makale, para politikası ile çevre kirliliği kontrolüne yönelik politikaların (düzenleyici politikalar) etkileşimini, üreticilerin nakit avans kısıtına tabi olduğu ve aynı zamanda üretim teknolojisi ve karbon emisyon oranları açısından farklılık gösterdiği bir genel denge modeli kullanarak karşılaştırmaktadır. Modeldeki çevresel unsurlar, üretime bağlı karbon emisyonu, emsiyonu azaltmayı amaçlayan teknoloji ve çevre vergisinden oluşmaktadır. Bu makaledeki analiz iki ana başlık etrafında düzenlenmiştir: Çevresel kaygıların varlığında para politikasının sosyal refaha nasıl etki ettiğinin araştırılması ve üretime bağlı çevre kirliliği ile çevresel düzenlemenin olduğu bir yapının para politikasına etkisi. Bulgular, ekonomideki aktörlerin, üretim teknolojisi, karbon emisyon oranları boyutlarında farklılık göstermesi durumunda, para politikasının sosyal refahın arttırılmasında ve kirliliğin azaltılmasına yönelik düzenleyici çabaların tamamlanması noktasında rol oynayabileceğini göstermektedir.

İkinci makale, işgücü piyasasında kısmi nakit avans kısıtı formunda nominal katılıkların olduğu bir ekonominin, farklı düzenleyici politikalar kapsamında verimlilik şoklarına tepkisini ve nominal katılığın derecesinin bu tepkiyi nasıl etkilediğini araştırmaktadır. Bu amaçla, işgücü piyasasında kısmi nakit avans kısıtı, üretim faaliyetiyle ilişkili çevre kirliliği ve kirliliği azaltmaya yönelik teknolojilerin olduğu stokastik bir genel denge modeli kullanılmaktadır. Çevre politikaları, fiyat politikası (vergi) ve piyasa mekanizması içeren miktar politikasından (cap-and-trade sytem) oluşmaktadır. Bulgular, fiyat düzenlemesi altında, makro değişkenlerdeki oynaklığın, miktar düzenlemesine kıyasla daha yüksek olduğunu göstermektedir. Miktar düzenlemesi kapsamında, çevre kirliliğini kontrol etmenin maliyeti üretim seviyesi ile pozitif olarak ilişkilidir. Bu kanal işgücünün ve diğer tüm değişkenlerin verimlilik şoklarına tepkisini azaltmaktadır. Ayrıca analizlerde nominal katılık derecesi arttıkça oynaklığın her iki düzenleme altında da arttığı, ancak fiyat düzenlemesinde artışın nispeten daha fazla olduğu gözlenmiştir.

Üçüncü makalede, sermaye girişi şokunun ekonomi dinamiklerine ve sosyal refaha etkisi farklı kontrol önlemleri altında araştırılmaktadır. Bu amaçla, ticarete konu olan sektörde yaparak öğrenme dışsallığı olan küçük açık ekonomi modeli kullanılmaktadır. Rekabetçi denge altında, modeldeki tüketim ve dış borçlanma sosyal olarak optimal miktarı aşmaktadır. Fiyat düzenlemesi kapsamında, düzenleyicinin rolü dış borçlanmaya vergi koymaktır. Miktar düzenlemesi altında ise düzenleyici, toplam dış borçlanmanın üst sınırı belirlenmekte ve bu miktarda borçlanma iznini, hanehalkının talep eden konumda olduğu spot piyasada arz etmektedir. Her iki düzenlemede de, düzenleyicinin politikayı sermaye şoku öncesinde belirlemesi nedeniyle bilgi asimetrisi söz konusudur. Ekonomideki diğer aktörler şoku gözlemledikten sonra karar almaktadır.

Üçüncü makale iki şekilde katkıda bulunmayı amaçlamaktadır: İlk olarak, fiyat ve miktar kontrollerini kıyaslayan yazından esinlenerek, sermaye girişleri için fiyat (vergi) ve piyasa mekanizmasını temel alan miktar tipi düzenlemenin refah etkilerini karşılaştırmaktadır. İkinci olarak, sermaye girişleri için piyasa mekanizmasına dayalı kontrol kavramını gündeme getirmektedir.

Üçüncü makalenin sonuçları, miktar düzenlemesi altında oynaklığın daha az olduğuna ve fayda açısından miktar politikasının kısa vadede fiyat politikasından daha iyi performans gösterdiğine işaret etmektedir. Bu üstünlük, sosyal refah fonksiyonunun şeklinden ve fiyat regülasyonu altında dış borcun değişkenlik göstermesinden kaynaklanmaktadır. Ayrıca, politikaların sıralaması, başlangıçtaki teknoloji düzeyinden etkilenmektedir: miktar kontrolü, başlangıçtaki teknoloji düzeyi düşük olduğunda fiyat kontrolüne kıyasla sosyal refah açısından daha iyi sonuç vermektedir. Ayrıca, fiyat politikasının miktar politikasına göre avantajı, teknoloji büyüme hızının artmasıyla birlikte azalmaktadır.

Anahtar Kelimeler: Fiyat ve Miktar Düzenlemeleri, Çevre Düzenlemeleri, Sermaye Kontrolleri

To my parents, for their love and support

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## LIST OF ABBREVIATIONS

- CIA : Cash-in-Advance
- DSGE : Dynamic Stochastic General Equilibrium
- IMF : International Monetary Fund
- NFA : Net Foreign Assets
- SMCE : Stationary Monetary Competitive Equilibrium
- TFP : Total Factor Productivity
- URR : Unremunerated Reserve Requirements

#### **CHAPTER I**

### **INTRODUCTION**

The discourse surrounding externalities, particularly environmental consideration, has long been a part of the research agenda of economists. External effects disrupt the competitive equilibrium and lead to Pareto inefficient outcomes. These effects are present when one agent's utility is directly affected by the choices made by another agent. Solutions to restore Pareto efficiency are well-documented in economics literature (Mas-Colell et al. 1995). These solutions encompass taxes, quotas, and market-based measures where the right to externality is traded. In the absence of uncertainty, taxes and quotas yield Pareto efficient allocations. Moreover, if externalities are well defined and enforceable property rights can be established, competitive decentralized markets for externality permits can also restore Pareto efficiency. Even the cap-and-trade system, which is concurrently used in practice for environmental control, is accurately defined in the literature as a partial market-based approach. Here, the government sets the total level of the externality and distributes tradable externality permits, each representing the right to generate one unit of externality. Through permit trading, the market reaches an efficient allocation.

While controlling for externalities is a mature field of research, the research on the interaction between macro policies and environmental policies is a relatively recent development. Over time, there has been a growing recognition of the significance of environmental control, paralleled by an increased awareness of the economy's impact on the environment. The acknowledgment of the economy's influence on the environment raises questions about whether macro policies should take account of these effects alongside environmental policies. Business cycles affect emissions and the design of environmental policy. Conversely, the design of environmental policies influences the dynamics of the economy in response to shocks. These interactions

raise questions about the optimal design of macroeconomic and environmental policies. Even if macro policies do not inherently aim to control pollution, the fact that environmental policies shape the economy's response to shocks suggests potential adjustments in macro policies to stabilize the system.

One important step in studying this interaction involves transitioning from comparing different regulatory policies, precisely price versus quantity controls, in a partial equilibrium context to analyzing them within a general equilibrium framework. Kelly (2005) addresses the optimal choice of regulation in the general equilibrium framework, demonstrating that the concavity of households' utility in consumption leads to general equilibrium effects favoring the quantity regulation. This approach results in less volatile production and consumption, aligning with households' desire for stability. Pizer (1999) also underscores the importance of designing optimal climate policies within a general equilibrium framework. This is important due to significant uncertainty surrounding the economy, such as productivity growth, which can influence future emissions.

A new strand of literature combines environmental economics and macroeconomics, aiming to understand the interactions between economy, economic policy, environment, and environmental policies in a Dynamic Stochastic General Equilibrium (DSGE) framework. Acknowledging the relationship between business cycles and emissions, Heutel (2012) investigate how environmental policy should optimally respond to business cycles within a DSGE model that includes a pollution externality in stock form. Their findings suggest that optimal regulatory policy adopts a procyclical approach, dampening the fluctuations in emissions. In a related study, Ramezani et al. (2020) explore the response of environmental policy to business cycles. They argue in favor of adapting environmental policies to macroeconomic fluctuations, proposing a pro-cyclical tax on emissions.

Using a DSGE model, Fischer and Springborn (2011) compare the dynamic effects on the economy of environmental policy choices (tax, cap on emissions, intensity target) under productivity shocks. Similarly, Annicchiarico and Di Dio (2015) investigate the economy's response to nominal and real shocks under alternative environmental regulations. Their DSGE model incorporates staggered price adjustment and pollution externality. They also study the optimal policy response to inflation under different environmental policy regimes. The study reveals that similar to Fischer and Springborn (2011), cap-and-trade mitigates volatility in key macroeconomic variables. Additionally, the degree of price stickiness affects the ranking of alternative regulations.

Dissou and Karnizova (2016) employ an environmental DSGE model to explore the impact of a cap-and-trade system and a carbon tax in the presence of sector specific productivity shocks. In another study, Annicchiarico and Di Dio (2017) explore the interaction of monetary policy with environmental policy in a New Keynesian model that incorporates pollution, abatement technology and environmental damage. This study delves into the impact of emission regulations on the business cycle, the influence of nominal rigidities on the macroeconomic effects of the environmental policy, and the optimal response of environmental policy to business cycles in the presence of nominal rigidity. The findings underscore that monetary policy influences the characterization of environmental policy and environmental concerns, in turn, affect the design of optimal monetary policy. The assumption of strict inflation targeting is no longer deemed optimal, as eliminating inefficiencies and boosting output may result in higher pollution. Overall research in this domain highlights the interactions between monetary and environmental policy, emphasizing that environmental concerns affect the design of optimal monetary policy.

The interaction of monetary policy and the environment is also studied using growth models. Faria (1998) and Faria et al. (2023) examine the impact of monetary policy on the environment within a growth model, treating the environment as a renewable asset (a form of capital) that contributes to both the utility function and the production process. Their findings suggest that money is environmentally non-neutral under cash-in-advance and transaction cost models.

The first two chapters of this thesis aim to contribute to this line of literature that studies the interaction of economy and environmental policies. In the second chapter, we explore the interaction of monetary policy and a regulatory policy for controlling pollution within an economy populated with financially constrained producers exhibiting heterogeneity in production technology and pollution rates. Our analysis is organized around two main topics: assessing the potential effect of monetary policy on social welfare in the presence of environmental concerns and investigating how the existence of pollution-type externality influences optimal monetary policy. To address these questions, we employ a heterogeneous agent Cash-in-Advance (CIA) Model based on Basci and Saglam (2005), coupled with a pollution externality framework outlined in Kelly (2005). Our findings suggest that in the presence of heterogeneity, monetary policy can enhance social welfare and complement regulatory efforts to mitigate pollution. We contribute to the existing literature by stressing the distributional impact of monetary policy. In our model with heterogeneity, monetary policy influences social welfare through production, affecting pollution and consumption. Monetary policy has no impact on abatement efforts. The impact of monetary policy on pollution is indirect, occurring through changes in the distribution of production. The direct impact on consumption operates through real wage adjustments and lump-sum money transfers, affecting consumption inequality. The indirect effect on consumption arises from the impact of monetary policy on optimal regulatory policy. Money growth redirects production away from the cash-constrained agents in the labor market. If cash-constrained agents also happen to be the more pollutant type, then this shift reduces overall emissions. This reduction creates room for a looser regulatory policy, leading to a higher overall consumption and, consequently, greater social welfare.

In the third chapter, we introduce uncertainty in the extended CIA model from the second chapter by adding total factor productivity shocks. In the stochastic version of

the model, the choice of regulatory framework, whether price or quantity-based, has different implications for model dynamics. This section compares price and quantity regulation for controlling pollution in a general equilibrium framework under the presence of nominal rigidities.

The chapter presents a stochastic general equilibrium model featuring nominal rigidity in the form of a partial cash-in-advance constraint in the labor market, pollution associated with production activity, and an abatement effort. Specifically, we investigate the impact of nominal rigidity on welfare under alternative regulatory policies. The primary question of interest revolves around how the system responds to productivity shocks under different regulatory policies and how the presence of nominal rigidity affects this response. Our key findings are twofold. First, volatility in macro variables is higher under price regulation compared to quantity regulation. Second, as the degree of nominal rigidity increases, volatility increases under both regulations but relatively more so under price regulation.

In the fourth chapter, we extend the discussion on price versus quantity controls to the realm of capital inflow control measures. This chapter aims to contribute in two ways: First, by drawing insights from the literature on price versus quantity controls, we compare the welfare implications of price and quantity-type regulation for capital inflows under uncertainty. Second, we introduce the concept of a market-based regulatory framework for capital inflows. While the effects of capital controls on welfare are theoretically studied, existing papers advocating the use of capital controls treat price-based and quantity-based controls as equivalent (Erten et al. 2021). However, as demonstrated by the literature on price versus quantity controls, the equivalence of these measures breaks down when the parameters of the economy are uncertain (Weitzman 1974). Only a few papers address the issue without making an analytic comparison (Erten et al. 2021; Ostry et al. 2011; Magud et al. 2011). For instance, Ostry et al. (2011) argue that price-based measures are easier to adjust cyclically but note that when authorities encounter information asymmetries and uncertainty re-

garding choices of the private sector, fixing the price-based measure to achieve the desired quantities can be challenging. According to the arguments in the same paper, quantity-based measures (administrative measures) are susceptible to rent-seeking behavior, and they should be used only if they can be made transparent and rule-based.

In theoretical frameworks, capital controls are often modeled as price-based measures. Recognizing the procyclical nature of global financial markets, prudential regulations on capital inflows that serve as countercyclical measures are considered justified (Gallagher et al. 2012; Korinek 2011). In other words, it is argued that for prudential purposes, tax on capital inflows should be procyclical, increasing during booms and decreasing during downturns (Davis et al. 2021; Aoki et al. 2016; Farhi and Werning 2014; Schmitt-Grohé and Uribe 2012). However, it is important to note that, in essence, price controls that are adjusted counter-cyclically along the business cycle are de facto equivalent to fixed quantity controls. A potential drawback of a tax compared to quantity restriction is that a small tax may not effectively deter massive inflows (Crotty and Epstein 1996). One recognized drawback of quantity-based measures is deficiency in transparency and susceptibility to rent-seeking behavior. However, a partial market-based approach combining quota and market mechanism is not prone to these concerns.

We introduce uncertainty over the global interest rate to the model developed by Benigno and Fornaro (2014, BF), which characterizes a small open economy experiencing endogenous growth and facing a financial resource curse. The model incorporates an externality in the form of households not internalizing the growth process in the tradable sector involving learning-by-doing. In the competitive equilibrium, this leads to consumption and external borrowing exceeding the socially optimal amount. Under the price regulation, the regulator's role is to set a tax on external borrowing. With quantity regulation, the regulator establishes a cap on aggregate external borrowing and issues borrowing permits. Households can buy these permits in the spot market where the government is the sole supplier. We compare the partial marketbased policy with the tax alternative. Under both regulations, there is information asymmetry as the regulator sets the policy before observing the interest rate. Agents, however, make decisions after observing the shock. The main questions of interest are: Which mode of regulation yields higher welfare? How does the ranking of policies depend on the initial level of technology (level of development) and the pace of technology growth? We conduct sensitivity analysis concerning these parameters, as the level of development is a key characteristic for categorizing countries.

We demonstrate that there is less volatility under quantity regulation and, in terms of utility, quantity policy outperforms price policy in the short-run. This superiority arises from the shape of the social welfare function and ex-post variation in external debt under price regulation. Given that the agents are risk averse and the social welfare is right skewed in external debt, the higher the ex-post variation in external debt, the greater the relative advantage of quantity over price policy in the short-term. Furthermore, the ranking of policies is influenced by the initial productivity level, where quantity control performs better in terms of social welfare when the initial productivity level is low. The relative advantage of price over quantity policy declines with an increase in the pace of technology growth.

Finally, Chapter V of the thesis summarizes the main findings of the previous chapters and discusses the thesis's contributions and potential avenues for further research.



#### **CHAPTER II**

## MONETARY AND ENVIRONMENTAL POLICY IN A CASH IN ADVANCE MODEL

#### 2.1. Introduction

The recognition of the importance of environmental control has evolved over time, with increasing awareness of the impact of the economy on the environment. In the literature on the environment, public policies addressing pollution primarily take the form of regulatory policy involving quantity limits and taxation (e.g. Weitzman 1974; Pizer 1999; Hoel and Karp 2002; Stavins 2019; Silva and Caplan 1997; Cremer et al. 2010). The role of monetary policy in the environmental context has been questioned to a lesser extent (Faria 1998; Faria et al. 2023; Annicchiarico and Di Dio 2015). In this paper, we study the interaction of monetary policy with regulatory policy in a setting involving pollution as a production externality and technology to partially contain it. Our analysis is organized around two main topics: the potential effect of monetary policy on social welfare in the presence of environmental concerns and how pollution-type externality influences optimal monetary policy. To address these questions, we combine Basci and Saglam (2005) heterogeneous agent Cash-in-Advance (CIA) Model with pollution externality as outlined in Kelly (2005).

Faria (1998) and Faria et al. (2023) examine the role of monetary policy in the environmental context through various monetary approaches, including a CIA model. These papers incorporate the environment as a stock variable to well-known monetary models, treating it as a renewable asset contributing to the utility function and the production process. In Faria (1998), the impact of money on the environment is explored within an extended Sidrauski's monetary growth model where money is an

argument of the representative agent's utility function.<sup>1</sup> This paper shows that if the utility function is not additively separable in consumption and real money balances, both money and inflation can influence the environment. However, the sign of this impact remains indeterminate. Similarly, Faria et al. (2023) investigate the impact of money and money growth on the environment across various models involving money. Their findings suggest that when money is introduced directly into the utility function, it is neutral. On the contrary, money is environmentally non-neutral under CIA and transaction cost models.

Environmental extensions in the models above are from the literature on environment and growth, conceptualizing the environment as a form of capital. We take a different approach by drawing from the literature on optimal control and model pollution as a production externality along with technology to control pollution. Unlike the models mentioned above, our framework, following Basci and Saglam (2005) features agents with different productivity and pollution levels that are cash-constrained in the labor and product markets. In this framework, the implication of money growth for production and consumption varies with respect to the type of agent. This structure enables us to account for the distributional impact of monetary policy.

The way we incorporate the environment in the model is closer in design to Annicchiarico and Di Dio (2015) where they study the interaction between monetary policy and the environment using a New Keynesian model that includes pollution, abatement technology, and environmental damage. They show that environmental concerns affect the design of optimal monetary policy.

In our model, money growth affects social welfare through both production thereby pollution- and consumption. The impact of monetary policy on pollution is indirect, occurring via the change in the distribution of production, and therefore is limited compared to a regulatory policy. Money growth shifts the production away

<sup>&</sup>lt;sup>1</sup>The extended model incorporates a state equation for the environment, with it serving as input in the production function.

from the cash-constrained agent in the labor market. There are both direct and indirect effects of monetary policy on consumption. The direct impact on consumption works through real wage adjustments and lump-sum money transfers, affecting consumption inequality. The indirect effect on consumption stems from the impact of monetary policy on optimal regulatory policy. As mentioned earlier, money growth shifts production away from cash-constrained agents in the labor market. If these agents also happen to be the more pollutant type, this shift reduces overall emissions and allows room for more loose regulatory policy. This results in higher overall consumption and, consequently, higher social welfare. The heterogeneity in agents' characteristics that result in consumption inequality and the heterogeneity in the pollution rates in our model is crucial in determining the role of monetary policy. Without it, given that there is already a regulatory policy designed to handle pollution, monetary policy has no impact on the environment and has no interaction with regulatory policy.

In summary, this chapter explores the interaction between regulatory and monetary policies for controlling pollution within an economy populated with financially constrained producers exhibiting heterogeneity in production technology, and the rate at which they pollute the environment. Our findings indicate that when there is heterogeneity, monetary policy has a role in improving social welfare and complementing regulatory efforts to address pollution.

The rest of the paper is organized as follows. Section 2.2 describes the model. Section 2.3 outlines the stationary monetary competitive equilibrium (SMCE). Section 2.4 describes the social planner's allocation, and Section 2.5 derives the optimal monetary and regulatory policy under SMCE. In Section 2.6, we present the outcomes of the numerical computations, illustrating the impact of alternative monetary and regulatory policy on the SMCE and examining how optimal monetary and regulatory policy respond to changes in the productivity levels, pollution rates, and the parameter of disutility derived from pollution. Finally, Section 2.7 concludes.

#### 2.2. The Model

#### 2.2.a. Environment

There are two types of infinitely lived agents indexed by i = 1, 2, who take the role of both consumer and producer. There exist  $N_i$  identical agents of type i, where  $N_i > 0$  for all i. Time is indexed by t. Labor is the only factor of production. Agents are endowed with  $\overline{L}_i$ . They do not value leisure. They produce the same good with a different technology,  $f_i(L_{it})$ . We assume that type 2 agents have superior technology, i.e.  $f'_2(L) > f'_1(L)$ , for all L > 0. Production functions have decreasing returns to scale technology. We further assume that  $\lim_{L\to 0} f'_i(L) = \infty$ . This assumption assures that both types of agents produce at the equilibrium.

There are no credit markets, and agents face cash-in-advance (CIA) constraints in labor and commodity markets. There is a pollution externality associated with production activity, with agents having partial control over the extent of pollution emitted. They are equipped with an abatement technology to convert a fraction of output into pollution control units as in Kelly (2005). These control units are represented as a concave function of the fraction of output reserved for pollution control. These control units reduce a fraction of pollution, while agents incur tax for the part of pollution that they do not control for.

The timing of the actions is important for the impact of monetary policy. Timing is as follows: Any period begins with monetary transfers. Then, the labor market opens and clears. Labor is hired, and production takes place. Next, pollution taxes are paid, and the tax revenues are transferred back to the agents. Later, the goods market opens and clears. Finally, the remaining stock of money is transferred to the next period.

#### 2.2.b. Agents' Problem

Representative agent of type *i*, who both produces and consumes, faces the following problem:

$$\max \sum_{t=0}^{\infty} \beta_i^t [u_i(c_{it}) - B_i(E_t)] \text{ subject to, for all } t$$

$$c_{it} = (1 - n_i) f_i \left( L_{it} + \overline{L}_i \right) + q_{it}$$
(2.1)

$$E_{it} = (1 - s_i(n_i))\gamma_i f_i \left( L_{it} + \overline{L}_i \right)$$
(2.2)

$$E_t = \sum_i N_i E_{it} \tag{2.3}$$

$$-\overline{L}_i \le L_{it} \le \frac{M_{it} + (i-1)(\alpha_i/N_i)M_t}{w_t}$$
(2.4)

$$-f_i(\overline{L}_i + L_{it}) \le q_{it} \le \frac{M_{it} + (\alpha_i/N_i)M_t - w_t L_{it} - \tau p_t E_{it} + T_i}{p_t}$$
(2.5)

$$M_{it+1} = M_{it} + (\alpha_i/N_i)M_t - w_t L_{it} - p_t q_{it} - \tau p_t E_{it} + T_i$$
(2.6)

$$M_{it+1} + (i-1)(\alpha_i/N_i)M_{t+1} \ge 0$$
(2.7)

$$M_{it+1} - w_{t+1}L_{it+1} + (\alpha_i/N_i)M_{t+1} \ge 0$$
(2.8)

Agents have a preference for consumption and receive disutility from aggregate pollution. We assume welfare to be additive in utility in consumption and disutility from pollution as in Kelly (2005). Utility from consumption  $u_i$  is increasing, strictly concave, and twice differentiable, and utility from pollution represented by  $B_i$  is increasing and convex. A fraction  $n_i$  of home product is used for pollution control. Per period consumption is the sum of home production net of output used for pollution control and purchases in the goods market (purchase if  $q_{it} > 0$ , sales if  $q_{it} < 0$ ) (eq. 2.1). We assume that pollution is a fraction of output measured by  $\gamma_i$  (eq. 2.2). We also assume that firms are endowed with technology to convert  $n_i$  units of output into  $s_i(n_i)$  pollution control units, in other words, scrubbers, as in (Kelly 2005). The technology that converts output into pollution control units is increasing and strictly concave, i.e.  $s'_i(n_i) > 0$  and  $s''_i(n_i) < 0$ . Labor is bounded below by endowment and bounded above by the money holdings of the agents that demand labor (eq. 2.4). Wages must be paid in advance of production activity. Sales are bounded below by quantity produced and bounded above by the money holdings of agents that demand goods (eq. 2.5). Before the goods market opens, money holdings consist of money transfers by the government, labor income, and net tax payments. Monetary holdings that remain after the goods market are transferred to the next period (eq. 2.6). Money holdings in the next period cannot be negative (eq. 2.7) and should be high enough to cover the advance payment for labor expenses (eq. 2.8).

#### 2.2.c. Government Behavior

The government has two roles in this economy: determining the money supply and setting the regulatory policy (tax policy) to control pollution. We assume that the economy starts with a positive stock of money  $M_0 = \sum_i N_i M_{i0}$ , where each type is borne with  $M_{i0}$  units of currency. Money stock at time *t* is denoted by  $M_t$  and evolves according to

$$M_{t+1} = (1+\alpha)M_t$$
, with  $\alpha > -1$ . (2.9)

Thus, money stock is always positive. The government allocates the injected money stock  $\alpha$  as a lump sum transfers so that each type *i* agent receives  $\alpha_i M_t / N_i$ , where  $\alpha_1 + \alpha_2 = \alpha$ .

The other role of the government is to regulate the production side of the economy to control the amount of pollution. For this purpose, the government can follow either a price (tax) or a quantity policy (cap). Without uncertainty, these two policies are equivalent (Weitzman 1974). However, if there is uncertainty, these policies, which

are determined ahead of activity, have different social welfare consequences. In this paper, there is no uncertainty, and we model as if the government conducts a regulatory tax policy. We assume that the government sets the tax  $\tau$  on emissions and agents choose the allocations. The government returns all tax revenue  $(TR_t)$  as lump-sum transfers to the agents  $(TR_{it}, i = 1, 2)$  in the economy:

$$TR_t = \tau p_t (N_1 E_{1t} + N_2 E_{2t}) = T_{1t} + T_{2t}$$
, for all t.

#### 2.2.d. Competitive Equilibrium

Here, we characterize the Stationary Monetary Competitive Equilibrium (SMCE). In the following sections, we solve for the social planner's outcome and optimal government policy under SMCE.

The set of prices and quantities  $\{p_t, w_t, \tau, c_{it}, L_{it}, q_{it}, n_i, M_{it+1} | i = 1, 2\}_{t=0}^{\infty}$  constitute a SMCE of the financially constrained production economy with a negative externality if,

- for each type of agent {c<sub>it</sub>,L<sub>it</sub>,q<sub>it</sub>,n<sub>i</sub>,M<sub>it+1</sub>}<sup>∞</sup><sub>t=0</sub> is solution to the constrained optimization problem of the agent under the sequence of
   {p<sub>t</sub>,w<sub>t</sub>,τ|p<sub>t</sub>,w<sub>t</sub>,τ<sub>t</sub> > 0}<sup>∞</sup><sub>t=0</sub>,
- 2. Real quantities  $\{c_{it}, L_{it}, q_{it}, E_{it} | i = 1, 2\}$  are constant over time,
- 3. Prices  $\{p_t, w_t\}$  increase at the rate of money growth  $\alpha$  for all t,
- 4. Labor market clears for all *t*, i.e.  $N_1L_{1t} + N_2L_{2t} = 0$ ,
- 5. Goods market clears for all t, i.e.  $N_1q_{1t} + N_2q_{2t} = 0$ ,
- 6. All revenue from pollution taxes is rebated to the agents, i.e.  $\tau p_t (N_1 E_{1t} + N_2 E_{2t}) = \tau p_t E_t = T_{1t} + T_{2t}$  for all *t*,
- 7. Money market clears for all t, i.e.  $N_1M_{1t+1} + N_2M_{2t+1} = M_{t+1}$ .

#### 2.2.e. Agents' Problem Redefined

Substituting for  $q_{it}$  from equation (2.6) in the consumption equation (eq. 2.1) and in cash-in-advance constraint for the goods market (eq. 2.5), the problem of agent *i* is redefined as:

$$\max \sum_{t=0}^{\infty} \beta_i^t [u_i(c_{i,t}) - B_i(E_t)] \text{ subject to, for all t}$$
(2.10)

$$c_{it} = (1 - n_i) f_i \left( L_{it} + \overline{L}_i \right) + \frac{M_{it} + (\alpha_i / N_i) M_t}{p_t} + \frac{T_i - \tau p_t E_{it}}{p_t} - \frac{w_t}{p_t} L_{it} - \frac{M_{it+1}}{p_t}$$
(2.11)

$$E_{it} = (1 - s_i(n_i))\gamma_i f_i(L_{it} + \overline{L}_i)$$
(2.12)

$$E_t = \sum_i N_i E_{it} \tag{2.13}$$

$$-\overline{L}_i \le L_{it} \le \frac{M_{it} + (i-1)(\alpha_i/N_i)M_t}{w_t}$$
(2.14)

$$0 \le M_{it+1} \le p_t f_i (L_{it} + \overline{L}_i) + M_{it} + (\alpha_i / N_i) M_t - w_t L_{it} - \tau p_t E_{it} + T_i$$
(2.15)

$$M_{it+1} + (i-1)(\alpha_i/N_i)M_{t+1} \ge 0$$
(2.16)

$$M_{it+1} - w_{t+1}L_{it+1} + (\alpha_i/N_i)M_{t+1} \ge 0$$
(2.17)

Given the set of prices  $\{p_t, w_t, \tau\}$ , and the set of choice variables  $\{L_{it}, n_i, M_{it+1}/p_t\}$ , i = 1, 2, the optimal allocation requires:<sup>2</sup>

$$\frac{\partial \mathscr{L}}{\partial L_{it}} \le 0, \quad L_{i,t} + \overline{L}_i \ge 0 \quad \text{and} \quad (L_{it} + \overline{L}_i) \frac{\partial \mathscr{L}}{\partial L_{it}} = 0 \tag{2.18}$$

<sup>&</sup>lt;sup>2</sup>Solution is provided in the Appendix.

$$\frac{\partial \mathscr{L}}{\partial (M_{it+1}/p_t)} \le 0, \quad \frac{M_{it+1}}{P_t} \ge 0 \quad \text{and} \quad \frac{\partial \mathscr{L}}{\partial M_{it+1}} \frac{M_{it+1}}{P_t} = 0$$
(2.19)

$$\frac{\partial \mathscr{L}}{\partial n_i} \le 0, \quad n_i \ge 0 \quad \text{and} \quad \frac{\partial \mathscr{L}}{\partial n_i} n_i = 0$$
 (2.20)

#### 2.3. Existence of a SCME

We will show that there exists a stationary equilibrium, where the more productive agent (type 2) buys labor from the less productive agent (type 1), i.e.  $L_{1t} < 0$  and  $L_{2t} > 0$ . Hence, cash-in-advance constraint in the labor market is binding only for type 2 agents. Type 2 agents transfer money to the next period with the only motivation to pay labor expenses in advance. Type 1 agents, on the other hand, have no motivation to transfer money to the next period, i.e.  $M_{1t+1} = 0$ . Due to the concavity of the utility function and the production function, both types of agents produce and consume at the equilibrium.

**Proposition 1** There is a stationary equilibrium where  $L_{1t} < 0$ ,  $L_{2t} > 0$ ,  $M_{1t+1} = 0$ , and  $M_{2t+1} > 0$  if the following conditions hold:

- 1.  $\beta_1 < 1 + \alpha$
- 2.  $\beta_2 \leq 1 + \alpha$
- 3.  $1 + \alpha \leq \beta_2 \delta_2 f'_2(\overline{L}_2) / \delta_1 f'_1(\overline{L}_1)$

where  $\delta_i = ((1 - n_i) - \tau(1 - s_i(n_i))\gamma_i)$  represents the portion of sales revenues that remains after controlling for pollution and paying emission tax.

Conditions in *Proposition 1* follow from the agents' utility maximization problem. If the first condition is violated, the type 1 agent would prefer to hoard money since transferring one unit of money to the next period yields a higher discounted utility than consuming today. The second condition ensures that the real wage rate is less than or equal to the marginal productivity of additional labor employed by the type 2 agent. Otherwise, he/she would prefer not to hire any labor. The last condition states that before the labor market opens, there is a possibility for trade as the initial marginal value of production for the type 1 agent is lower than the marginal value of production for type 2 agent. Hence, the type 1 agent has an incentive to supply labor, and the type 2 agent has an incentive to demand labor.

SCME, whenever exists, satisfies (2.21)-(2.31):

$$\frac{\beta_2 \delta_2 f'_{2t} (L_{2t} + \overline{L_2})}{1 + \alpha} = \omega_t = \frac{w_t}{p_t} = \delta_1 f'_{1t} (L_{1t} + \overline{L}_1)$$
(2.21)

$$\delta_i = ((1 - n_i) - \tau (1 - s_i(n_i))\gamma_i) \text{ for } i = 1,2$$
(2.22)

$$s_i'(n_i) = \frac{1}{\tau \gamma_i} \tag{2.23}$$

$$L_{2t} = -\frac{N_1 L_{1t}}{N_2} \tag{2.24}$$

$$v_t = \frac{(1+\alpha_2)M_t/N_2}{L_{2t}}$$
(2.25)

$$p_t = \frac{1}{\delta_2 \beta_2} \frac{w_t}{f'_{2t}} (1 + \alpha)$$
(2.26)

$$p_t q_{it} = -w_t L_{it} \frac{1+\alpha}{1+\alpha_2} - \tau p_t E_{it} + T_i$$
(2.27)

$$c_{it} = (1 - n_i)f_i \left( L_{it} + \overline{L}_i \right) - \frac{w_t}{p_t} L_{it} \frac{1 + \alpha}{1 + \alpha_2} + \frac{T_{it} - \tau p_t E_{it}}{p_t}$$
(2.28)

$$\tau p_t (N_1 E_{1t} + N_2 E_{2t}) = T_{1t} + T_{2t}$$
(2.29)

$$M_{10} = \mu M_0, \quad \mu = 0 \tag{2.30}$$

$$M_{2t+1} = M_{2t}(1+\alpha) \tag{2.31}$$

At the equilibrium, each agent sets the demand/supply of labor such that the marginal product of labor is equal to the real wage rate (eq. 2.21). Type *i* agents attempt to cope with the pollutants and reduce output revenues by a factor of  $\delta_i$ . Firms

would operate only if revenue after pollution control is greater than zero. Furthermore, the concavity of  $s_i(n_i)$  guarantees that  $\delta_i < 1$ , therefore  $\delta_i \in (0, 1)$ . Moreover,  $\delta_i$  is decreasing in  $\tau$ . Taking the derivative of  $\delta_i$  w.r.t.  $\tau$  we get:

$$\frac{\partial \delta_{i}}{\partial \tau} = \underbrace{\frac{\partial n_{i}}{\partial \tau}(-1 + \tau \gamma_{i} s_{i}^{'}(n_{i}))}_{=0} - \gamma_{i}(1 - s_{i}(n_{i})) < 0.$$
(2.32)

Equation (2.23) addresses the optimal choice of the rate of pollution control. In the equilibrium, the amount of output set aside for pollution control is such that the marginal benefit of controlling pollution, in other words, the real amount that is not foregone as taxes at the margin  $\tau \gamma_i s'_i(n_i)$  is equal to the real marginal cost of controlling pollution, corresponding real price of the consumption good, which is one. Equation (2.24) follows from the labor market equilibrium. The nominal wage is determined by the cash-in-advance constraint (eq. 2.25). Price is deduced from the expression for the real wage rate (eq. 2.26). Purchases/sales are functions of labor income and net tax revenues (eq. 2.27). As demonstrated in *Proposition 1*, type 1 agents do not transfer money between periods; this also holds for the initial period (eq. 2.30). Money holdings of the type 2 agent increase at the rate of total money growth (eq. 2.31). Notice that labor allocation depends on both regulatory and monetary policy (eq. 2.21). However, the decision on  $n_i$  only depends on the regulatory policy. In what follows, we define how real wage and the allocation of labor change with respect to the pollution tax rate and the money growth rate.

# **Corollary 1.1** *Higher tax: (i) reduces labor used by the agent that is more pollutant, and (ii) reduces real wage.*

Notice that labor's response to an increase in the tax rate is conditional on the relative emission rates; however, the response of the real wage rate is definite, and it declines no matter which agent is more pollutant.

**Proof.** The Proof is in the Appendix. All the detailed proofs are relegated to the appendix for the rest of the paper.

**Corollary 1.2** An increase in the money growth rate  $\alpha$  reduces the equilibrium real wage and shifts production towards the less productive agent.

#### 2.4. Social Planner's Problem

Social welfare (*SW*) is the aggregate utility at a given time, represented by equation (2.33) below. The government sets the allocation of labor ( $L_{it}$ , i = 1, 2), the rate of pollution control ( $n_i$ , i = 1, 2) and the distribution of total output among agents ( $\psi$ , where  $\psi \in (0, 1)$ ) such that social welfare is maximized. Hence, the government faces the following problem,

$$\underset{\psi, n_{i}, L_{it}}{Max} SW = \sum_{i=1}^{2} N_{i} [u_{i}(c_{it}) - B_{i}(E_{t})] \text{ subject to,}$$
(2.33)

$$c_t = \sum_{i=1}^{2} N_i (1 - n_i) f_i \left( L_{it} + \overline{L}_i \right)$$
(2.34)

$$c_{1t} = \psi c_t / N_1 \tag{2.35}$$

$$c_{2t} = (1 - \psi)c_t / N_2 \tag{2.36}$$

$$E_{it} = (1 - s_i(n_i))\gamma_i f_i \left(L_{it} + \overline{L}_i\right), \text{ for } i = 1, 2$$
(2.37)

$$E_t = \sum_{i=1}^2 N_i E_{it}, \quad \sum_{i=1}^2 N_i L_{it} = 0.$$
 (2.38)

We assume social welfare to be additive in utility in consumption and disutility in pollution as in Kelly (2005). The total consumption is equal to the total production net of the amount reserved for pollution control (*eq*.2.34). In the above characterization of the social planner's problem, we assume that the government can distribute total output in any alternative way { $\psi \in (0,1)$ }. Pollution is proportional to output. The social planner's solution is given in *Proposition 2*.

**Proposition 2** Under the social planner's solution:

- The optimal allocation of consumption requires  $u'_1(c_{1t}) = u'_2(c_{2t})$ .
- The optimal fraction of output n<sub>i</sub> satisfies the following equation,

$$B'\gamma_i s'_i(n_i) = u', \tag{2.39}$$

where 
$$u' = \psi u'_1(c_{1t}) + (1 - \psi)u'_2(c_{2t})$$
 and  $B' = N_1B'_1 + N_2B'_2$ .

• The optimal labor allocation satisfies the following equation,

$$\left((1-n_1)u' - (1-s_1(n_1))\gamma_1 B'\right)f_1'\left(L_{1,t} + \overline{L}_1\right) = \left((1-n_2)u' - (1-s_2(n_2))\gamma_2 B'\right)f_2'\left(L_{2,t} + \overline{L}_2\right).$$
(2.40)

Notice that even if disutility from externality is different for each type of agent  $(B_i)$ , when utility is additive, socially optimal allocation calls for equality in marginal utilities in consumption, i.e. optimal distribution of total output among agents satisfies  $u'_1(c_{1t}) = u'_2(c_{2t})$ . This result directly follows from the maximization of the social welfare function with respect to  $\psi$ . If the social planner has the power to distribute the total output in a socially optimal way, then  $u' = u'_1(c_{1t}) = u'_2(c_{2t})$ . Otherwise, if the distribution of output is given, then u' is the weighted average of marginal utilities.

Condition in equation (2.39) states that in the social planner's equilibrium  $n_i$  is set such that the marginal utility from consuming one more unit of the consumption good is equal to the marginal disutility of pollution. The optimal labor allocation is such that the marginal increase in social welfare due to allocating one more unit of labor to a type 1 agent is equal to that of type 2 agent. Here, the marginal increase in social welfare is the marginal utility of consumption net of marginal disutility from pollution. **Corollary 2.1** Optimality condition in equation (2.40) indicates that, (i) if there is no pollution, then it is optimal to manage activity such that  $f'_1 = f'_2$ , (ii) if there is pollution but no heterogeneity in the pollution rate, then it is again optimal to set  $f'_1 = f'_2$ , (iii) however when there is heterogeneity in the pollution rate ( $\gamma_i$ ), the type of agent with higher (lower) pollution rate produces less (more) compared to the allocation where  $f'_1 = f'_2$ .

## 2.5. Optimal Government Policy Under Competitive Equilibrium

Under the competitive equilibrium, optimal regulatory and monetary policies depend on the assumptions about how the pollution tax revenues are redistributed among agents. If it is possible to redistribute tax revenues in a socially optimal way such that in the equilibrium marginal utilities from consumption are equal for different types of agents, then it is possible to achieve the social planner's equilibrium by regulatory and monetary policies. In this case, regulatory policy is designed to control pollution, and monetary policy is conducted with the objective of overcoming the frictions caused by the cash-in-advance constraint, and the optimal money growth is determined as if there is no pollution externality.

Under the competitive equilibrium, the government maximizes SW as in equation (2.33) w.r.t.  $\tau$  and  $\alpha$  subject to (2.21)-(2.31). Accordingly, optimal  $\tau$  and  $\alpha$  require that the following conditions are satisfied:

$$N_1 u_1' \frac{\partial c_{1t}}{\partial \tau} + N_2 u_2' \frac{\partial c_{2t}}{\partial \tau} - \frac{\partial E_t}{\partial \tau} = 0$$
 (2.41)

$$N_1 u_1' \frac{\partial c_{1t}}{\partial \alpha} + N_2 u_2' \frac{\partial c_{2t}}{\partial \alpha} - \frac{\partial E_t}{\partial \alpha} = 0$$
 (2.42)

Using the equations (2.35) and (2.36) these conditions simplify as the following:

$$(u_{1t}' - u_{2t}')c_t \frac{\partial \psi}{\partial \tau} + (\psi u_{1t}' + (1 - \psi)u_{2t}')\frac{\partial c_t}{\partial \tau} - B'\frac{\partial E}{\partial \tau} = 0$$
(2.43)

$$(u'_{1t} - u'_{2t})c_t \frac{\partial \Psi}{\partial \alpha} + (\Psi u'_{1t} + (1 - \Psi)u'_{2t})\frac{\partial c_t}{\partial \alpha} - B'\frac{\partial E}{\partial \alpha} = 0$$
(2.44)

The conditions given in equations (2.43) and (2.44) clearly show that if there is inequality in consumption, the impact of government policies on income distribution needs to be considered when determining the optimal policy. In that case, the optimal government policy is not only about maximizing total output and minimizing pollution, but it is also about redistributing income. The following proposition describes the optimal regulatory and monetary policy under two different cases: one where the government does have the means to redistribute tax revenues in a socially optimal way and another where the government has no such mechanism.

**Proposition 3** (i) If the redistribution of tax revenues in a socially optimal way is possible, i.e.  $u'_1 = u'_2 = u'$ , then we can obtain social planner's outcome under the competitive equilibrium with optimal regulatory tax and money growth given as,

$$\tau = \frac{B'}{u'} \tag{2.45}$$

$$\beta_2 = (1 + \alpha). \tag{2.46}$$

(ii) if the redistribution of tax revenues in a socially optimal way is possible, optimal monetary policy is not independent of regulatory policy and other structural parameters of the model.

In the first case, optimal tax rate and optimal money growth are set independently. The optimal tax rate is increasing in disutility from pollution and is decreasing in utility from consumption. And the optimal money growth is pinned down by the discount rate of the more productive agent. It is not affected by the existence of a pollution externality.

In the second case, optimal monetary policy is not independent of the regulatory policy and other structural parameters of the model. To show this point, we consider one special case where the tax revenues are not distributed in a socially optimal way. Assuming that the collected tax from each agent is returned as lump-sum transfer, i.e.  $T_{it} = \tau p_t E_{it}$ , then optimal tax rate and optimal money growth should satisfy the following equations.

$$\frac{B'}{\tau} = \frac{\left(\frac{N_1}{N_2}\frac{\partial n_1}{\partial \tau}\frac{f_{1t}}{f_{2t}}u'_{1t} + \frac{\partial n_2}{\partial \tau}u'_{2t}\right)}{\left(\frac{N_1}{N_2}\frac{\partial n_1}{\partial \tau}\frac{f_{1t}}{f_{2t}} + \frac{\partial n_2}{\partial \tau}\right)} + \frac{(1-s_2)\gamma_2\left(\frac{(1+\alpha)}{(1+\alpha_2)}\right)\left(u'_{1t} - u'_{2t}\right)}{\left(\frac{f_{2t}}{f'_{2t}}\frac{1}{L_{2t}}\right)\frac{(1+\alpha)}{\beta_2}\left(\frac{N_1}{N_2}\frac{\partial n_1}{\partial \tau}\frac{f_{1t}}{f_{2t}} + \frac{\partial n_2}{\partial \tau}\right)}$$
(2.47)

$$\frac{(1+\alpha)}{\beta_2} = \frac{\frac{\left(u'_{1t}(1-n_1)-B'(1-s_1)\gamma_1\right)}{\delta_1}}{\left(\frac{u'_{2t}(1-n_2)-B'(1-s_2)\gamma_2}{\delta_2}\right) + \left(\frac{\beta_2}{1+\alpha_2}\right)\left(u'_{1t}-u'_{2t}\right)\left(1+\frac{f''_{2t}L_{2t}}{f'_{2t}}\right)}$$
(2.48)

There is no closed-from solution, and we must rely on numerical computation for comparative statistics for uncovering the optimal government policy. In the next part, we do simulation exercises, first to evaluate the impact of regulatory and monetary policy on the SMCE and then to uncover optimal money growth and the tax rate under the alternative parametrization of the model.

## 2.6. Numerical Experiments

In this section, we carry out two types of simulation exercises. First, we compute the competitive equilibrium and social welfare under alternative monetary and regulatory policies. Second, in the subsequent simulation exercise, we compute optimal monetary and regulatory policy in response to variations in the productivity of the type 1 agent, the pollution rate of the type 2 agent, and the degree of disutility from pollution. For the simulation exercise, we assume the following functional forms:

$$s_i(n_i) = 1 + \kappa \frac{n_i^{1-\varepsilon_i}}{1-\varepsilon_i} \text{ for } i = 1,2$$
(2.49)

$$u_i = 1 + \frac{c_{it}^{1-\sigma_i}}{1-\sigma_i}$$
 for  $i = 1, 2$  (2.50)

$$f_i(L_{it} + \overline{L}_i) = A_i(L_{it} + \overline{L}_i)^{\lambda_i} \text{ for } i = 1,2$$

$$(2.51)$$

## 2.6.a. Impact of Government Policy on SMCE and Social Welfare

In this exercise, we compute the equilibrium under alternative monetary and regulatory policies, assuming the parameter set given in Table 2.1. In particular, we are doing a grid search over different values of  $\alpha$  and  $\tau$ . We aim to illustrate model's properties. The exploration of the model in a more realistic framework is deferred to future research. Therefore, values in Table 2.1 are not calibrated to match any economic fact; rather they are set such that assumptions of the model are satisfied. In particular,  $\varepsilon_i$  and  $\kappa$  are set to ensure that the abatement technology is strictly concave in the share of output reserved for pollution control.

Notation	Description	Value
γ1	0.6	Pollution rate of type 1 agent
$\gamma_2$	0.8	Pollution rate of type 2 agent
$\beta_2$	0.98	Discount factor of type 2 agent
$\lambda_1$	0.25	Output elasticity of type 1 agent
$\lambda_2$	0.375 (0.25*1.5)	Output elasticity of type 2 agent
$\sigma_1 = \sigma_2$	2	Relative risk aversion
$B^{'}$	0.01	Disutility from pollution
$\alpha_1$	α	Share of money stock allocated to type 1 agent
$\alpha_2$	0	Share of money stock allocated to type 2 agent
$\overline{L}_1, \overline{L}_2$	10, 0	Labor endowments
$A_1$	1	Total factor productivity of type 1 agent
$A_2$	2.5	Total factor productivity of type 1 agent
$\varepsilon_1 = \varepsilon_2$	2	Parameter setting the curvature of the abatement technology
κ	1/50	Constant in the abatement technology

Table 2.1. Parameter Set for the Numerical Experiments in Chapter II

Under this parametrization of the model, in the equilibrium, the type 1 agent consumes less then the type 2 agent, e.i.  $c_{1t} < c_{2t}$ . We are assuming that money growth takes place as lump-sum transfers to agent 1. Together with the functional forms given in equations (2.49) to (2.51), following equations characterize the equilibrium:

$$n_i = (\tau \gamma_i \kappa)^{(1/\varepsilon_i)} \text{ for } i = 1,2$$
(2.52)

$$\delta_i = ((1 - n_i) - \tau (1 - s_i(n_i))\gamma_i) \text{ for } i = 1,2$$
(2.53)

$$f_i(L_{it} + \overline{L}_i) = A_i(L_{it} + \overline{L}_i)^{\lambda_i} \text{ for } i = 1,2$$

$$(2.54)$$

$$\frac{\beta_2 \delta_2 f'_{2t}(L_{2t})}{1+\alpha} = \delta_1 f'_{1t} \left(-\frac{N_2}{N_1} L_{2t} + \overline{L}_1\right)$$
(2.55)

Equation (2.49) explicitly defines the technology for converting output into pollution control units. Combining equation (2.49) with the equilibrium condition (2.23), we get the expression for the share of output that is set aside for pollution control (eq. 2.52). It is an increasing function of the tax rate and the pollution rate. Equation (2.51) represents the production function. Equation (2.53) is the proportion of revenues left after pollution control and tax payments, and (2.55) is the same as the equilibrium condition already defined in equation (2.21). Equation (2.54) rewrites equation (2.51).

Figures 2.1 and 2.2 display equilibrium responses to alternative tax rates and money growth rates. Table 2.2 below summarizes the outcomes. A higher tax rate has a negligible impact on total output but reduces total consumption, as a greater portion of the output is set aside for pollution control. With a higher tax rate, production shifts from more pollutant agent 2 to less pollutant agent 1, causing a decline in real wage as demonstrated in Corollary 1.1. Both agent's consumption declines, but due to erosion in real wages, consumption of the first agent is affected more, worsening consumption inequality. As the tax rate increases, disutility from pollution declines at the expense of lower overall consumption and worsening consumption inequality. Due to this trade-off, social welfare increases initially in response to higher taxes but decreases after a certain point. The response of social welfare to the tax rate follows a hump-shaped curve, indicating an interior solution characterizing optimal regulatory policy.

Figure 2.2 shows that when  $c_{1t} < c_{2t}$ , increasing money growth through lump-sum transfers to agent 1 can improve social welfare. In response to higher money growth, production shifts from the more productive agent (type 2) to the less productive agent

	$f_1$	$f_2$	ω	$c_1$	<i>c</i> <sub>2</sub>	$c_1/c_2$	SW
Higher $ au$	$\checkmark$	X	$\searrow$	$\searrow$	$\searrow$	$\searrow$	$\searrow$
Higher $\alpha$	$^{\mathbf{k}}$	X	$\searrow$	$\nearrow$	$\searrow$	$\nearrow$	$\searrow$

 Table 2.2. Simulation Outcomes: Response to Policy

	Total consumption	Total emission	Consumption inequality
Higher $ au$	$\searrow$	$\searrow$	7
Higher $\alpha$	$\searrow$	$\searrow$	$\searrow$

 Table 2.3. Summary of Simulation Outcomes: Response to Policy

(type 1), leading to a decline in real wage as in Corollary 1.2. While overall consumption declines, the consumption of type 1 agents increases due to monetary transfers. Notably, higher money growth affects social welfare through three channels. First, the decline in total output and shift in production away from the more productive type reduces total output and total consumption. Second, conjointly with output, emissions decline. Third, overall consumption declines, but consumption of the first agent increases due to monetary transfers. Consequently, higher money growth increases consumption of the type that consumes less, improving consumption inequality. In this case, the distributional impact of monetary policy dominates, and initially, social welfare to monetary policy is hump-shaped. The optimal money growth exceeds the rate that maximizes total output in the absence of pollution externality, which is set by the equality  $1 + \alpha = \beta_2$ .

In summary, regulatory and monetary policies affect social welfare through three channels: overall consumption, total emissions, and consumption inequality. Table 2.3 summarizes the simulation results regarding these channels. Higher tax and higher money growth reduce total consumption and total emissions. Lower emissions come at the expense of lower consumption. However, the implications of regulatory and monetary policies for consumption inequality are different. While higher tax rates increase consumption inequality, higher money growth rates reduce it.

**Remark 1** Given the assumptions on the concavity of the utility function, production

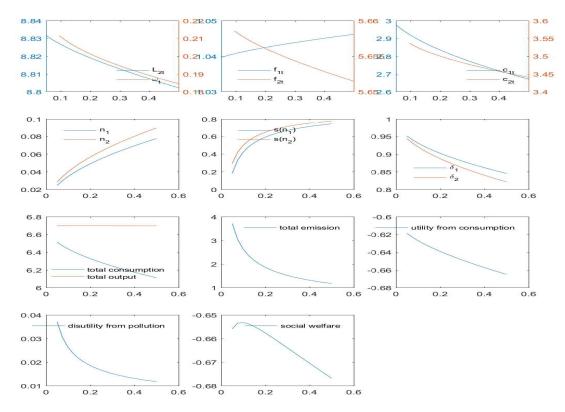
function and the technology for producing control units, the response of social welfare to the tax rate follows a hump-shaped curve, with the optimal tax rate determined as an interior solution.

A higher tax rate reduces pollution at the cost of lower consumption. For low levels of tax rates, the gains from reduced pollution outweigh the reduction in the utility from consumption, resulting in an increase in social welfare. However, as the tax rate increases beyond a certain point, this trend reverses, and the declining utility from consumption surpasses the gains from lower pollution. A higher tax rate also decreases the consumption of type 1 agent relative to type 2 agent.

**Remark 2** The response of social welfare to monetary policy is hump-shaped. Higher money growth worsens financial rigidities, reduces total output and overall consumption but improves consumption inequality. This shift in consumption limits the drop in social welfare. Therefore, there are parametrizations where higher money growth initially increases total utility from consumption and improves social welfare. Optimal money growth is determined as an interior solution. Some positive money growth is deemed desirable.

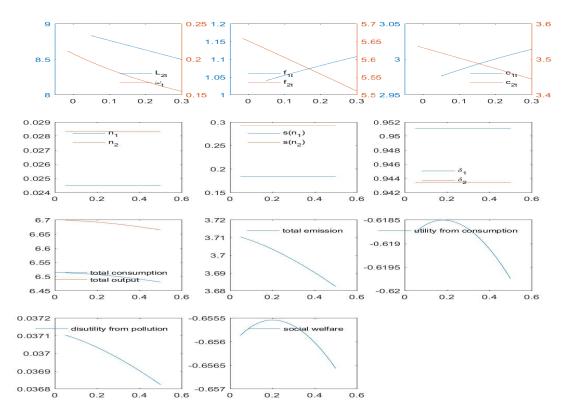
**Remark 3** The impact of monetary policy on pollution is limited compared to the tax policy, as it influences total emissions through production shifts between agents. However, monetary policy has pronounced constructive distributional effects on consumption.

# Figure 2.1. Equilibrium Under Alternative $\tau$



x-axis: au (ightarrow higher tax rate)

**Figure 2.2.** Equilibrium Under Alternative  $\alpha$ 



x-axis:  $\alpha$  ( $\rightarrow$  higher money growth)

## 2.6.b. Optimal Regulatory and Monetary Policy

The objective of this simulation exercise is to jointly determine optimal regulatory and monetary policy in response to variations in the productivity of agent 1, relative pollution intensity of agent 2, and the disutility parameter. We solve for the optimal monetary and regulatory policy under different parametrizations for  $\lambda_1$ ,  $\gamma_2$  and B', satisfying the following equations simultaneously.

$$n_i = (\kappa \tau \gamma_i)^{1/(\varepsilon_i)} \tag{2.56}$$

$$\delta_i = (1 - n_i) - \tau (1 - s_i(n_i)) \gamma_i \text{ for } i = 1, 2$$
(2.57)

$$\frac{\beta_2 \delta_2 A_2 \lambda_2 L_{2t}^{\lambda_2 - 1}}{1 + \alpha} = \delta_1 A_1 \lambda_1 (-L_{2t} + \overline{L}_1)^{\lambda_1 - 1}$$
(2.58)

$$\frac{\partial n_i}{\partial \tau} = (-1/\tau) \frac{s_i'(n_i)}{s_i''(n_i)} = \frac{n_i}{\tau \varepsilon_i}$$
(2.59)

$$\frac{f_1}{f_2} = \frac{A_1(-\frac{N_2}{N_1}L_{2t} + \overline{L}_1)^{\lambda_1}}{A_2 L_{2t}^{\lambda_2}}$$
(2.60)

$$c_{1t} = (1 - n_1)A_1 \left(-\frac{N_2}{N_1}L_{2t} + \overline{L}_1\right)^{\lambda_1} + \delta_1 A_1 \lambda_1 \left(-\frac{N_2}{N_1}L_{2t} + \overline{L}_1\right)^{\lambda_1 - 1} \frac{N_2}{N_1}L_{2t} \frac{1 + \alpha}{1 + \alpha_2} \quad (2.61)$$

$$c_{2t} = (1 - n_{2t})A_2L_{2t}^{\lambda_2} - \delta_1A_1\lambda_1(-\frac{N_2}{N_1}L_{2t} + \overline{L}_1)^{\lambda_1 - 1}L_{2t}\frac{1 + \alpha}{1 + \alpha_2}$$
(2.62)

$$\frac{B'}{\tau} \left( \frac{N_2}{N_1} \frac{\partial n_{1t}}{\partial \tau} \frac{f_1}{f_2} + \frac{\partial n_{2t}}{\partial \tau} \right) = \left( \frac{N_2}{N_1} \frac{\partial n_1}{\partial \tau} \frac{f_1}{f_2} u'_1 + \frac{\partial n_2}{\partial \tau} u'_2 \right) + (2.63)$$

$$(1 - s_2) \gamma_2 \lambda_2 \left( \frac{\beta_2}{1 + \alpha_2} \right) \left( u'_1 - u'_2 \right)$$

$$(1+\alpha)\left(\frac{\left(u_{2}^{'}(1-n_{2})-B^{'}(1-s_{2})\gamma_{2}\right)}{\beta_{2}\delta_{2}}+\frac{\left(u_{1}^{'}-u_{2}^{'}\right)\lambda_{2}}{1+\alpha_{2}}\right) \qquad (2.64)$$
$$= \frac{\left(u_{1}^{'}(1-n_{1})-B^{'}(1-s_{1})\gamma_{1}\right)}{\delta_{1}}$$

Equations (2.56) to (2.58) are as in the simulation exercise in the previous part. Equation (2.59) is the derivative of  $n_i$  with respect to the tax rate. It is derived by differentiating equation (2.23) w.r.t.  $\tau$ . Equation (2.60) is the ratio of the output of type 1 agent to type 2 agent. Equations (2.61) and (2.62) are consumption of agent 1 and 2 after substituting for  $f_i$  and  $f'_i$ . Equations (2.63) and (2.64) jointly determine the optimal tax rate and the optimal money growth. They are simplified versions of equations (2.47) and (2.48).

Simulation results are presented in Figures 2.3 to 2.5.

**Remark 4** *The higher productivity of type 1 agent (relative productivity of agent 2 to agent 1 being fixed) reduces the optimal money growth and increases the optimal tax rate (Figure 2.3).* 

Higher productivity implies higher real wage, reducing consumption inequality and improving social welfare. This, in turn, enables for a relatively higher tax rate and lower money growth.

**Remark 5** A higher parameter for disutility from emissions increases both the optimal money growth and the optimal tax rate (Figure 2.4). This time, it is desirable to bring down overall production, and more so of the more pollutant type. Under the optimal policies, production by the second agent declines, while production by the first agent increases slightly. Consumption declines for both types of agents, as output is used for pollution control, resulting in reduced emissions.

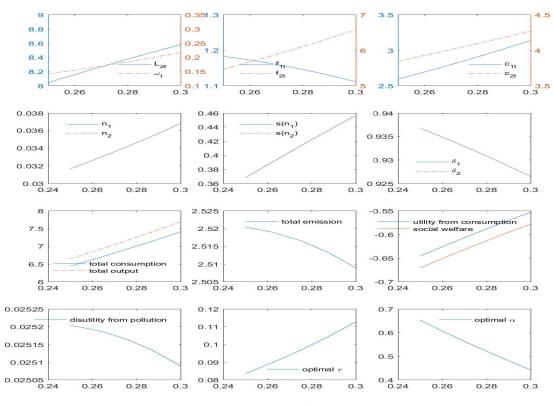
**Remark 6** *The more pollutant is agent 2, the higher is the optimal money growth, and the lower is the optimal tax rate (Figure 2.5).* 

Higher money growth creates a space for regulatory policy to be looser. The higher pollution intensity of agent 2 is counteracted by higher inflation. Higher money growth induces a shift in production towards a less pollutant sector, allowing the optimal tax rate to decline. This results in an increase in consumption for both types

of agents. The presence of monetary policy enables the optimal tax rate to be set at a lower level.

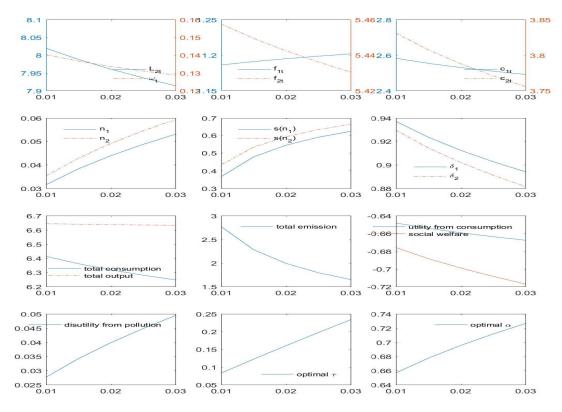


**Figure 2.3.** Optimal Regulatory and Monetary Policy Under Alternative  $\lambda_1$ 



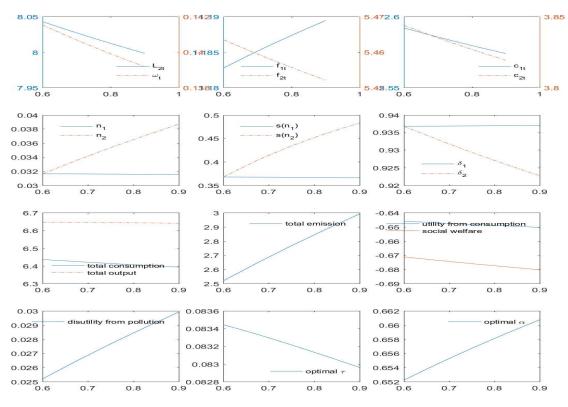
x-axis:  $\lambda_1^{}$  (ightarrow higher productivity)

## **Figure 2.4.** Optimal Regulatory and Monetary Policy Under Alternative B'



x-axis: Br ( $\rightarrow$  higher disutility due to unit emission)

## Figure 2.5. Optimal Regulatory and Monetary Policy Under Alternative $\gamma_2$



x-axis:  $\gamma_{\rm 2}~(\rightarrow {\rm higher~pollution})$ 

## 2.7. Conclusion

In this paper, we study the interaction between regulatory and monetary policies for controlling pollution in an economy populated with financially constrained producers that are heterogeneous concerning production technology and the rate at which they pollute the environment. We show that, in the absence of mechanisms to equalize consumption across agents, monetary policy has a role in improving social welfare and complementing regulatory initiatives in taking action for pollution control.

In the model setup, it is important to note that monetary policy does not directly influence abatement; rather, its impact on pollution control is indirect. The rate of money growth affects the allocation of production between sectors. Furthermore, monetary policy has a dual effect on consumption - a direct impact through subsidizing the type of agent that consumes less, thereby alleviating consumption inequality and increasing overall welfare, and an indirect effect by shifting production away from the more pollutant sector. This shift allows for lower tax on emissions, contributing to increased consumption for both types of agents.

The interaction between monetary policy, regulatory policy, and the environment is a young field of research. This chapter contributes to this literature by providing an alternative modeling framework for studying optimal policies. In the next chapter, we introduce uncertainty into this model and compare the model dynamics under alternative environmental policies, namely, the price and quantity controls.



## **CHAPTER III**

# PRICE VS. QUANTITY CONTROLS IN A PARTIAL CASH-IN-ADVANCE MODEL

## **3.1. Introduction**

Environmental regulations have long been a part of the research agenda of economists, and recently, they have also been integrated into macro policy research. The interplay between monetary policy, environmental policy, the macroeconomy, and the environment has been explored using Dynamic Stochastic General Equilibrium (DSGE) Models that incorporate elements such as nominal rigidity, pollution as a negative production externality, and an abatement technology to partially contain pollutant emissions. In Chapter II, following this line of research, we extend a heterogeneous agent cash-in-advance model to incorporate pollution externality. This extension aims to investigate the potential role of monetary policy in controlling environmental pollution and exploring the repercussions of environmental regulations on monetary policy.

In this chapter, we introduce uncertainty in the same model by adding total factor productivity shocks. This chapter presents a stochastic general equilibrium model featuring nominal rigidity in the form of a partial cash-in-advance constraint in the labor market, pollution associated with production activity, and an abatement effort. The main question of interest is how the system responds to productivity shocks under alternative environmental policies and how the scale of nominal rigidity affects this response. Environmental policies encompass price and quantity (cap-and-trade) regulation for pollution control.

In the stochastic version of the model, the choice between a price or quantitybased regulatory framework yields distinct implications for model dynamics and welfare. Additionally, the scale of the cash-in-advance constraint affects model dynamics at varying magnitudes under each regulatory policy. Using the framework in our model, we can also compare the impact of nominal rigidity on volatility under different regulatory policies aimed at controlling pollution.

This paper is associated with two research areas. To begin with, it is linked with the literature that compares price versus quantity controls in a general equilibrium framework (e.g. Kelly 2005; Pizer 1999). In addition, it is part of the literature exploring the interaction between monetary policy and environmental policy (e.g. Heutel 2012; Annicchiarico and Di Dio 2015). Section 3.2. provides a selected survey of these research fields.

Our main findings are as follows. First, volatility in macro variables is higher under price regulation compared to quantity regulation. Second, as the degree of nominal rigidity increases, volatility rises under both regulations, but this increase is relatively more pronounced under price regulation. Furthermore, we demonstrate that the stochastic cash-in-advance model is unstable if producers face full cash constraints in the labor market. Our results align with those of Annicchiarico and Di Dio (2015), who compare the macroeconomic implications of alternative environmental policies, including a cap on emissions and a tax policy using a New Keynesian-type DSGE model embodying pollutant emissions. They also observe that emission caps dampen the response of macroeconomic variables to shocks and that higher nominal rigidity increases volatility under both environmental policy regimes.

The rest of this chapter is organized as follows. Section 3.2. briefly reviews the price versus quantity controls literature. Section 3.3. outlines the heterogenous agent cash-in-advance model, which is extended to include environmental regulation. This section provides a separate description of agents' problem under both price and quantity regulations. In Section 3.4 we present the solution of the stochastic model. In Section 3.5., we characterize the non-stochastic monetary competitive equilibrium. In Section 3.6., we explore the system's response to a persistent productivity shock us-

ing log-linearized versions of the model. In this part, after log-linearizing the system, we demonstrate that it is unstable under full cash-in-advance constraint. Furthermore, we compute and compare the theoretical moments of the main macro variables in the model under both regulatory frameworks. In Section 3.7., we compute a quadratic approximation to expected social welfare and carry out numerical exercises comparing volatility under two regulatory frameworks with respect to volatility and social welfare. We also compare the response of volatility and social welfare to changes in the degree of nominal rigidity under different regulatory policies. Finally, Section 3.8. concludes.

## 3.2. Selected Literature on Price vs. Quantity Controls

This line of literature explores the optimal means of regulating a variable. The main question revolves around whether to establish a control mechanism directly through quantities or indirectly by employing prices as instruments. In a seminal paper, Weitzman (1974) argued that price and quantity controls produce the same equilibrium allocation under full information. This duality breaks down in the presence of uncertainty regarding the costs and benefits associated with the regulated economic variable.<sup>3</sup> In situations involving uncertainty, optimal policy depends on the slopes of the marginal benefit and marginal cost schedules. In more formal terms, originally put forward by Weitzman (1974), cost and benefit are expressed as a second-order approximation around the optimal quantity ( $\hat{q}$ ). They are represented by:

$$C(q,\theta) = a(\theta) + (C' + \alpha(\theta))(q - \hat{q}) + \frac{C''}{2}(q - \hat{q})^2$$
(3.2)

$$B(q,\eta) = b(\eta) + (B' + \beta(\eta))(q - \hat{q}) + \frac{B''}{2}(q - \hat{q})^2$$
(3.3)

<sup>&</sup>lt;sup>3</sup>Stavins (2019) thoroughly discusses the similarities and differences of tax and cap-and-trade approaches from the perspective of controlling for carbon emissions.

where  $a(\theta)$ ,  $\alpha(\theta)$ ,  $b(\eta)$ ,  $\beta(\eta)$  are stochastic functions and C', C'', B', B'' are fixed coefficients. There is uncertainty about the level of the marginal cost and benefit curves and no correlation between the uncertainty of cost and benefit. Optimal quantity instrument  $\hat{q}$  is solution to the following optimization problem:

$$\max_{q} E\left[B(q, \eta) - C(q, \theta)\right]$$
(3.4)

If instead price is the control instrument, price is announced, and quantity adjusts to the price  $(h(p, \theta))$ . Let  $\tilde{p}$  be the solution to the following optimization problem:

$$\max_{p} E\left[B(h(p,\theta),\eta) - C(h(p,\theta),\theta)\right]$$
(3.5)

The expected comparative advantage of prices over quantities becomes,

$$\Delta = E\left[\left(B((\tilde{q}(\theta)), \eta) - C((\tilde{q}(\theta)), \theta)\right) - (B(\hat{q}, \eta) - C(\hat{q}, \theta))\right].$$
(3.6)

Once we solve the model and substitute in for the expressions for cost and benefit under alternative modes of control, we end up with the following simplified expression:

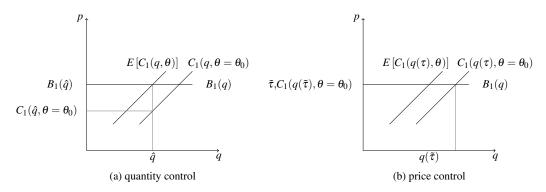
$$\Delta = \frac{\sigma_{\theta}^2}{2C''^2} (C'' + B'')$$
(3.7)

which states that price (quantity) control works better when marginal benefits are relatively flat (steep). The intuition behind this result is as follows. A sharply curved benefit function implies that agents are risk-averse, exhibiting a heightened distaste for volatility in quantities. A nearly flat marginal cost schedule indicates that an inaccurately determined price would result in a more substantial deviation of output from the desired quantity. In such cases, it is advisable for the regulator to directly set the quantity and let the price fluctuate. Further elaboration on this issue is provided in the subsequent discussion below.

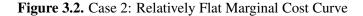
The optimal policies, whether based on price or quantity, are established ex-ante to

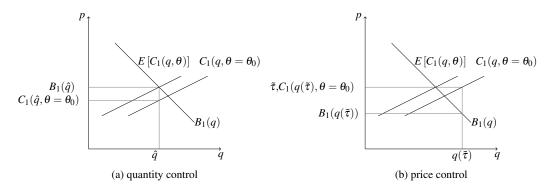
ensure that expected marginal benefit and marginal cost are equal. However, following the realization of the shock, ex-post marginal benefit and marginal cost no longer align. Therefore, the preferred policy is the one that minimizes the gap between expost marginal cost and marginal benefit. The magnitude of this gap is contingent on the shapes of the cost and benefit schedules. Figures 3.1 and 3.2 provide a comparison of price and quantity policies for two distinct cases: one where the marginal benefit is flat (case 1) and another where the marginal cost curve is nearly flat (case 2). For ease of interpretation, we assume no uncertainty over the benefit schedule, i.e.,  $\eta$  is constant. In other words, the marginal benefit schedule remains fixed and does not shift. The optimal quantity  $(\hat{q})$  and the optimal price  $(\tilde{\tau})$  are identified as the point where the expected marginal cost  $E[C_1(q, \theta)]$  equals the expected marginal benefit  $(B_1(q))$ . The marginal cost schedule after the shock realization, with  $\theta = \theta_0$  is represented by  $C_1(\hat{q}, \theta = \theta_0)$ . In situations where the marginal benefit curve is relatively flat (figure 3.1), price policy outperforms quantity policy. Given a flat marginal benefit curve, under price policy, the marginal benefit remains equal to the marginal cost even after the shock, while under quantity control, the marginal cost falls below marginal benefit at  $\hat{q}$ . When the marginal cost curve is relatively flat (figure 3.2), the quantity policy outperforms the price policy. In this scenario, the gap between marginal benefit and marginal cost is lower under quantity control.

Figure 3.1. Case 1: Flat Marginal Benefit Curve



On this topic, one line of research developed by modifying the assumptions in





Weitzman's analysis. Laffont (1977) introduces further uncertainty by assuming that not only the levels but also the slopes of the marginal cost and benefit curves are random. This extra uncertainty favors the quantity regulation more. Yohe (1978) adds random disturbance to output, assuming that output can vary even under quantity regulation. Stavins (1996) shows that a positive correlation between environmental cost and benefit shocks (as opposed to the no correlation assumption in Weitzman (1974)) is more likely to favor quantity regulation.

Another line of research extends the initial contribution by incorporating it into a dynamic policy context with intertemporal quantity trading and policy updating. In a two-period setting with policy updating and cost uncertainty revealed in the second period, Pizer and Prest (2020) show that quantity regulation traded over time is superior to price regulation in maximizing social welfare. Intertemporal quantity trading allows firms to postpone or bring forward production options between periods. This tradability, also referred to as the option to bank and borrow, links the firm's actions over two periods. Under this setup, the firm can deduce policy in the second period before acting in the first period through the knowledge of benefit and cost parameters and the regulator's predetermined updating rule. Firms, having complete information on the costs and the regulator's behavior, can optimally adjust in all periods. This behavioral linkage, coupled with the absence of uncertainty for the regulator in the second period, enables quantity controls to achieve the first best outcome in both periods.

The heightened focus on environmental issues has revived interest in Weitzman's work. This research field focuses on situations where environmental damages are stock rather than flow. Defining the externality as a stock transforms the regulatory problem into a dynamic one. Weitzmans' basic intuition still holds in these models, accompanied by additional issues. When environmental damages are conceptualized as a stock, the timing of the cost and benefits becomes important for the regulatory policy. It necessitates a balance between current costs and future benefits (Hoel and Karp 2002). Considering future benefits introduces discount and stock decay rates as important factors. Furthermore, the ranking of policies is contingent on the method of policy implementation concerning the possibility of adjustment over time. Two distinct approaches are considered: an open-loop policy, where the regulator announces the entire policy trajectory initially, and a feedback policy, where the regulator can adjust the policy as new information arrives. In the feedback case, the ability to learn about costs and the capability to act upon them affect the ranking between taxes and quotas. In a setup where the regulator employs an open-loop policy, a lower discount rate or lower stock decay rate tends to favor quantity measures (Staring 1995). Newell and Pizer (2003) consider an open-loop policy where costs are serially correlated, and show that a more positive serial correlation favors the use of quotas. Karp and Zhang (2005) explore feedback policy with correlated shocks. By observing quota trading and firms' response to the tax, the regulator gains insight into the random value of the shock. They compare open-loop and feedback policies with and without quota trading when shocks are serially correlated, providing criteria for ranking taxes and quotas for the control of stock pollutants. When the regulator uses tax or quotas that can be traded, the regulator benefits from the information that he derives from firms' response to the tax under price regulation or price of quotas under quantity regulation. Under the feedback policy, he can condition his policy on this information.

A significant advancement has been the treatment of price versus quantity comparison in a general equilibrium setup. Pizer (1999) introduces uncertainty in a framework of optimal policy design for climate change utilizing an intertemporal model of the economy and climate. This paper uses a modified stochastic growth model to capture optimal consumer behavior, considering numerous correlated shocks. Within this model, uncertainties extend various parameters including the discount rate, the risk aversion of the consumer, elasticity of output with respect to capital, productivity growth, variability in productivity growth, and the depreciation rate of capital. The findings indicate a preference for taxes over quantities across the assumed range of values in the model. This preference arises because marginal damage is relatively flat and potentially negatively correlated with marginal costs. As the authors admit, results considerably depend on the functional forms, specifications, and parametrization. The overarching message is clear: uncertainty matters. Pizer (1999) also stresses the importance of designing optimal climate policies within a general equilibrium framework. This approach accounts for significant uncertainty surrounding the economy, such as productivity growth, which profoundly influences future emissions and the valuation of of future returns through interest rates.

Kelly (2005) investigates price and quantity regulation in general equilibrium when the regulator faces uncertainty about the firm's productivity shocks. The findings reveal that the comparative advantage of quantity versus price control in general equilibrium is not solely determined by the slopes of marginal benefits and costs; it is also influenced by general equilibrium effects, such as effects arising due to the concavity of households' utility in consumption. Under price regulation, the regulated variable is an increasing function of productivity shocks, leading to a higher variation in production. In the context of risk-averse households, increased variation reduces welfare. General equilibrium effects favor quantity control. As households' utility exhibits greater concavity, indicating a preference for smoother consumption, the attractiveness of quantity regulation grows. The preference stems from the implication that quantity regulation results in less volatile production and consumption, aligning with households' desire for stability. Acknowledging the relationship between business cycles and emissions, Ramezani et al. (2020) undertake a comparative analysis of the environmental effects of fixed and flexible taxes on emissions in response to a transitory technology shock. The investigation is done within a real business cycle model framework, shedding light on adapting environmental policies to macroeconomic fluctuations. The study concludes that policymakers should take an effective environmental policy capable of managing emission fluctuations. The authors argue in favor of a pro-cyclical tax regime, emphasizing that this approach incentivizes firms to maintain abatement efforts. Otherwise, firms lose motivation to make abatement efforts. Another benefit of variable tax policy is that it can respond to changes in the marginal value of consumption.

A new strand of literature combines environmental economics and macroeconomics aiming to understand the interactions between economy, economic policy, environment and environmental policies (e.g. Fischer and Springborn 2011; Heutel 2012; Annicchiarico and Di Dio 2015; Angelopoulos et al. 2010, 2013). Fischer and Springborn (2011) use a dynamic stochastic general equilibrium (DSGE) model to compare economy's response to productivity shocks under different policies for containing pollution. These policies encompass an emissions cap, an emissions tax, and an intensity target (maximum emissions to output ratio). Results indicate that under a same constraint on emission reduction, total output is highest under the intensity target policy compared to other policies. The cap policy yields the lowest volatility. The tax policy exhibits the highest volatility, accompanied by also highest production and utility.

Heutel (2012) investigates how environmental policy optimally responds to business cycles within a DSGE model, including a pollution externality in stock form. This paper compares a static regulatory policy with a dynamic one that optimally adapts to persistent productivity shocks. The model generates two offsetting effects in response to a productivity shock: a positive productivity shock increases welfare, leading to a higher demand for a cleaner environment and lower emissions. However,

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more productive capital also increases the opportunity cost of abatement, bringing about lower demand for abatement and higher emissions. The latter effect (price effect) dominates the first effect (income effect), making it optimal to have procyclical emissions. Notably, the optimal regulatory policy also adopts a procyclical approach, effectively dampening the fluctuations in emissions.

Annicchiarico and Di Dio (2015) study the economy's response to nominal and real shocks under alternative environmental regulations using a DSGE model that incorporates staggered price adjustment and pollution externality. Environmental regimes covered include cap-and-trade, emission intensity target (an exogenous limit on emission per unit of output) and a tax policy. The study reveals that, akin to findings in Fischer and Springborn (2011), cap-and-trade mitigates volatility in main macroeconomic variables. Moreover, the degree of price stickiness affects the ranking of alternative regulations. If prices are less sticky mean welfare is higher under tax policy, alternatively if prices adjust slowly then mean welfare is higher under a cap regime. Optimal environmental tax response is also influenced by the degree of adjustment in prices and the reaction of monetary policy to fluctuations.

Dissou and Karnizova (2016) employ an environmental DSGE model to explore the impact of a cap-and-trade system and a carbon tax given sector specific productivity shocks. The study, calibrated to the US economy, compares these alternative regulations based on volatility and welfare, measured by consumers' utility. The difference between the two instruments becomes apparent mainly in the case of shocks to energy production. Despite that the tax policy yields higher volatility of macroeconomic variables than the cap, it is more favorable in terms of welfare. Annicchiarico and Di Dio (2017) study the interaction of monetary policy with environmental policy in a New Keynesian model that incorporates pollution, abatement technology, and environmental damage. In particular, the study delves into the impact of emission regulations on the business cycle, the influence of nominal rigidities on the macroeconomic effects of the environmental policy, and the optimal response of environmental policy to business cycles in the presence of nominal rigidity. Additionally, the study explores how monetary policy affects optimal environmental policy. Various governance situations are considered, where the planner controls both the environmental and monetary policy, controls only monetary policy taking environmental policy as given, or controls only environmental policy given monetary policy. The findings highlight that monetary policy affects the characterization of environmental policy, and environmental concerns affect the optimal monetary policy design; the optimality of strict inflation targeting no longer holds.

Annicchiarico and Diluiso (2019) take the discussion of price versus quantity controls into a DSGE model featuring two interdependent economies. They compare the implications of two strategies for controlling emissions: a carbon tax and a cap-andtrade scheme, where trade in emission permits between countries is possible. They show that the propagation of economic shocks across countries is affected by the regulatory framework in use.

The first two chapters of this thesis aim to contribute to this evolving field, investigating the interaction between nominal rigidities, monetary policy, and regulatory policy in a cash-in-advance model extended to account for pollution externality and abatement technology. These chapters add to the exploration of potential interactions between regulatory policies and the macroeconomics.

## 3.3. Model

This chapter's framework is a slightly modified version of the model introduced in Chapter II. We introduce uncertainty to the extended heterogeneous agent Cash-in-Advance (CIA) Model by adding productivity shocks.

#### 3.3.a. Environment

The environment of the model is like the one presented in Chapter II. However, two main differences are introduced in this case. First, we incorporate productivity shocks. Additionally, we assume a partial cash-in-advance constraint in the labor market instead of a full constraint. This adjustment allows us to study the implications of change in the scale of the cash constraint.

In this stochastic version of the model, we introduce productivity shocks as in Kelly (2005). These shocks affect total factor productivity. Notably, there is information asymmetry between the firms and the regulator regarding the uncertain component of productivity. While the regulator cannot observe the shock, agents possess full information.

There are two types of infinitely lived agents indexed by i = 1, 2, each playing the roles of both consumer and producer. There exist  $N_i$  identical agents of type i, where  $N_i > 0$  for all i. Time is indexed by t. Labor is the only factor of production. Agents are endowed with  $\overline{L}_i$ , and they do not value leisure. The agents produce the same good but with different technologies,  $e^{z_{it}} f_i(L_{it} + \overline{L}_i)$ , where  $z_{it}$  represent shocks affecting total factor productivity of agent i. We assume that type 2 agents have superior technology, i.e.  $e^{z_{2t}} f'_2(L) > e^{z_{1t}} f'_1(L)$ , for all L > 0 and under all distributions of shocks. Production functions exhibit decreasing returns to scale technology. We further assume that  $\lim_{L\to 0} f'_i(L) = \infty$ . This assumption ensures that both types of agents produce at the equilibrium.

There are no credit markets, and agents face cash-in-advance constraints in labor and commodity markets. In the labor market, only a portion of the total wage payments must be made in advance. Additionally, there is a pollution externality associated with production activity, with agents having partial control over the extent of pollution emitted. They are equipped with an abatement technology to convert a fraction of output into pollution control units as in Kelly (2005). These control units are represented as a concave function of the fraction of output reserved for pollution control. They effectively reduce a fraction of pollution, while agents incur a cost for the part of pollution that they do not control for. This cost is in the form of tax payments under the price regulation and expenses for purchasing pollution permits under quantity regulation.

#### 3.3.b. Government

The government plays a dual role in this economy: it determines the money supply and establishes the regulatory policy to control pollution. We assume that at the outset, there is a positive stock of money  $M_0 = \sum_i N_i M_{i0}$ , where each agent of type *i* is borne with  $M_{i0}$  units of currency. Money stock at time *t* is denoted by  $M_t$  and evolves according to

$$M_{t+1} = (1+\alpha)M_t$$
, with  $\alpha > -1$ . (3.8)

Thus, money stock is always positive. The government allocates the injected money stock  $\alpha$  as a lump sum transfers so that each type *i* agent receives  $\alpha_i M_t / N_i$ , where  $\alpha_1 + \alpha_2 = \alpha$ .

The other role of the government is to implement environmental regulations. For this purpose, the government can adopt either a price (tax) or a quantity (cap on emissions) policy. In the absence of uncertainty, these two policies are equivalent . However, when uncertainty is present, these policies, which are determined ahead of activity, have different social welfare consequences (Weitzman 1974).

Under price control, the government sets a fixed real price  $p_t^e/p_t$  per unit of pollution emissions. Under quantity control, the government sets a cap on total emission  $(\overline{X})$  and supplies an equal amount of emission permits inelastically in the spot market. In this market, agents are buyers, and the price  $p_t^e$  adjusts to bring supply and demand into balance. Agents are required to own permits that cover their pollution emissions. These permits expire at the end of the period. Hence, the only motive for agents to hold these permits is to meet regulatory requirement for that period alone.

#### 3.3.c. Agents' Problem Under Price Regulation

Agents in this model are both consumers and producers. Representative agent of type *i* faces the following problem:

$$\max_{L_{it},\frac{M_{it+1}}{p_t},n_i} \sum_{t=0}^{\infty} \beta_i^t (u_i(c_{it}) - B_i(E_t)) \text{subject to, for all t}$$

$$c_{it} = (1 - n_i)e^{z_{it}} f_i \left(L_{it} + \overline{L}_i\right) + q_{it} - (1 - \theta)\frac{w_t L_{it}}{p_t}$$
(3.9)

$$E_{it} = (1 - s_i(n_i))\gamma_i e^{z_{it}} f_i \left( L_{it} + \overline{L}_i \right)$$
(3.10)

$$\sum_{i} N_i E_{it} = E_t \tag{3.11}$$

$$-\overline{L}_i \le L_{it} \le \frac{M_{it} + (i-1)(\alpha_i/N_i)M_t}{\theta w_t}$$
(3.12)

$$-(1-n_i)e^{z_{it}}f_i(\overline{L}_i + L_{it}) \le q_{it} \le \frac{M_{it} + (\alpha_i/N_i)M_t - \theta_{w_t}L_{it} - p_t^e E_{it} + T_{it}}{p_t}$$
(3.13)

$$M_{it+1} = M_{it} + (\alpha_i / N_i) M_t - \theta_{w_t} L_{it} - p_t q_{it} - p_t^e E_{it} + T_{it}$$
(3.14)

$$M_{it+1} + (i-1)(\alpha_i/N_i)M_{t+1} \ge 0$$
(3.15)

$$M_{it+1} - \theta w_{t+1} L_{it+1} + (\alpha_i / N_i) M_{t+1} \ge 0$$
(3.16)

Subscripts *i* and *t* denote the type of agent and time, respectively. Agents have a preference for consumption and incur disutility from aggregate pollution. We assume that the utility of agent *i* from consumption  $u_i$  is increasing, strictly concave, and twice differentiable. Additionally, the utility of agent *i* from pollution, represented by  $B_i$ , is increasing and convex. Per period consumption is a combination of home production, net of output used for pollution control, purchases in the goods market (with purchase if  $q_{it} > 0$  and sales if  $q_{it} < 0$ ) and in-kind payments made after production (eq. 3.9). In-kind payments amount to  $(1 - \theta)$  portion of real wage payments. The production process involves uncertainty, with the production function expressed as

 $e^{z_{it}} f_i (L_{it} + \overline{L}_i)$ , where  $z_{it}$ , i = 1, 2 represents shocks affecting total factor productivity of agent *i*.

Pollution is a fraction  $(\gamma_i)$  of output (eq. 3.10). Firms are equipped with technology to convert  $n_i$  units of output into  $s_i(n_i)$  pollution control units, represented as scrubbers as in Kelly (2005). This technology is increasing and strictly concave in  $n_i$ , i.e.  $s'_i(n_i) > 0$  and  $s''_i(n_i) < 0$ .

Labor is bounded below by endowment. An agent cannot supply more labor than he/she is endowed with. Labor is bounded above by the money holdings of the agents demanding labor. Part of wages (by a fraction of  $\theta$ ) must be paid in advance of production activity (eq. 3.12). Sales in the goods market are bounded below by quantity produced net of the amount reserved for pollution control and bounded above by the money holdings of agents demanding goods (eq. 3.13).

Money holdings before the goods market include money transfers by the government, labor income, and net tax payments. Money holdings that remain after the goods market closure are transferred to the next period (eq. 3.14). Money holdings for the next period cannot be negative (eq. 3.15) and must be sufficient to cover advance payments for labor expenses (eq. 3.16). Agents must have enough money, including transfers, to cover their labor expenses next period.

## 3.3.d. Agents' Problem Under Quantity Regulation

Representative agent of type *i* faces the following problem:

$$\max_{X_{it},L_{it},\frac{M_{it+1}}{p_t},n_{it}} \sum_{t=0}^{\infty} \beta_i^t (u_i(c_{it}) - B_i(E_t)) \text{subject to, for all t}$$

$$c_{it} = (1 - n_{it})e^{z_{it}}f_i\left(L_{it} + \overline{L}_i\right) + q_{it} - (1 - \theta)\frac{w_t L_{it}}{p_t}$$
(3.17)

$$E_{it} = (1 - s_i(n_i))\gamma_i e^{z_{it}} f_i \left( L_{it} + \overline{L}_i \right) \le X_{it}$$
(3.18)

$$\sum_{i} N_i E_{it} = \sum_{i} N_i X_{it} = \overline{X}_t \tag{3.19}$$

$$-\overline{L}_i \le L_{it} \le \frac{M_{it} + (i-1)(\alpha_i/N_i)M_t}{\theta_{W_t}}$$
(3.20)

$$-(1 - n_{it})e^{z_{it}}f_i(\overline{L}_i + L_{it}) \le q_{it} \le \frac{M_{it} + (\alpha_i/N_i)M_t - \theta_{w_t}L_{it} - p_t^e X_{it}}{p_t}$$
(3.21)

$$M_{it+1} = M_{it} + (\alpha_i / N_i) M_t - \theta_w L_{it} - p_t q_{it} - p_t^e E_{it}$$
(3.22)

$$M_{it+1} + (i-1)(\alpha_i/N_i)M_{t+1} \ge 0$$
(3.23)

$$M_{it+1} - \theta w_{t+1} L_{it+1} + (\alpha_i / N_i) M_{t+1} \ge 0$$
(3.24)

The model's structure under quantity regulation deviates from the setup with price regulation, particularly in terms of how the regulatory framework is defined. Quantity regulation takes the form of a cap and trade system. In this framework, the regulator sets a cap on total emissions, denoted by  $\overline{E}_t$ , and supplies permits inelastically at the amount  $\overline{X}_t = \overline{E}_t$  in the spot market. Agents demand permits  $X_{it}$ , and the price  $p_t^e$ adjusts to bring supply and demand into balance. Ultimately, each agent must have the amount of permit  $X_{it}$  covering its pollution emission (eq. 3.18). In this setup,  $X_{it}$ is an asset.

#### 3.4. Solution of the Stochastic Model

Maintaining the basic assumptions of the non-stochastic model presented in proposition 1 in Chapter II, we can solve the stochastic model. Equations (3.25)-(3.36) characterize a dynamic system under productivity shocks. Under both regulations, the same equation set determines the dynamics, with the difference in interpretation: under price regulation, equation (3.29) is just an identity that defines aggregate emissions, whereas, under quantity regulation, it defines the equilibrium of the market in permits. The real price of permits  $p_t^e/p_t$  is fixed under price control, whereas under quantity regulation, it is a function of output, which is time-varying.

$$\frac{\frac{w_t}{w_{t+1}}\delta_{2t+1}e^{z_{2t+1}}f'_{2t+1}}{\frac{\theta u'_{2t}}{\beta_2 u'_{2t+1}} + (1-\theta)\frac{p_t}{p_{t+1}}} = \frac{w_t}{p_t} = f'_{1t}e^{z_{1t}}\delta_{1t}$$
(3.25)

$$\delta_{it} = ((1 - n_{it}) - \frac{p_t^e}{p_t} (1 - s_i(n_{it}))\gamma_i) \text{ for } i=1,2$$
(3.26)

$$s'_{it}(n_{it}) = \frac{1}{(p_t^e/p_t)\gamma_i}$$
 (3.27)

$$E_{it} = (1 - s_i(n_{it}))\gamma_i f_{it} \text{ for } i = 1,2$$
(3.28)

$$\sum_{i} N_{i} E_{it} = E_{t} \text{ under price regulation}$$

$$\sum_{i} N_{i} E_{it} = \sum_{i} N_{i} X_{it} = \overline{X}_{t} = E_{t} \text{ under quantity regulation}$$
(3.29)

$$L_{2t} = -\frac{N_1 L_{1t}}{N_2} \tag{3.30}$$

$$w_t = \frac{(1 + \alpha_2)M_t/N_2}{\theta L_{2t}}$$
(3.31)

$$p_t = \frac{w_t}{\delta_{1t} e^{z_{1t}} f_{1t}'}$$
(3.32)

$$p_t q_{it} = \frac{\alpha_i}{N_i} M_t - \theta w_t L_{it} \frac{1 + \alpha}{1 + \alpha_2} + \frac{T_{it} - \tau_t E_{it}}{p_t}$$
(3.33)

$$c_{it} = (1 - n_{it})f_{it} - \frac{w_t}{p_t}L_{1t}\frac{1 + \alpha_2 + \theta\alpha_1}{1 + \alpha_2} + \frac{T_{it} - \tau_t E_{it}}{p_t}$$
(3.34)

$$M_{1t} = 0 \text{ and } M_{2t} = M_t \tag{3.35}$$

$$M_{2t+1} = M_{2t}(1+\alpha) \tag{3.36}$$

Under the presence of shocks, both real variables and the growth rate of nominal variables are no longer constant. Therefore, the optimality condition that comes from setting marginal productivity equal to real wage becomes an intertemporal equation linking labor across consecutive periods (eq. 3.25). Given the labor supply in the current period  $L_{1t}$  and the shocks  $z_{it}$  and  $z_{it+1}$  one can deduce the labor supply in the subsequent period, denoted as  $L_{1t+1}$ . Pollution control reduces revenues by a factor

of  $\delta_{it}$  (eq. 3.26). Under price control  $p_t^e/p_t$  is fixed, so are  $\delta_{it}$ ,  $n_{it}$ , and  $s_{it}$ . While, under quantity control,  $p_t^e/p_t$  responds to output, therefore  $\delta_{it}$ ,  $n_{it}$ , and  $s_{it}$  are time varying. Equation 3.27 defines the optimal choice of pollution control,  $n_i$ , equalizing the real marginal cost and benefit of controlling pollution. The part of pollution that is not controlled for is the pollution emitted (eq. 3.28). Equation (3.30) follows from the labor market equilibrium. The nominal wage is determined by the cash-inadvance constraint (eq. 3.31). The price of the consumption good is derived from the expression for the real wage (eq. 3.32). Notably, both wage and price levels are influenced by the productivity shocks. Purchase and sales are a function of money transfers, labor income, and net tax revenues (eq. 3.33). Following Proposition 1, type 1 agents do not transfer money to the next period; this also applies to the initial period (eq. 3.35). Money holdings of the type 2 agents increase at the rate of total money growth (eq. 3.36).

The model has a non-stochastic steady state, which we will define in the following section.

#### 3.5. Non-Stochastic Steady State

Non-stochastic steady state, with  $z_{it} = 0$  for all t and i = 1, 2, corresponds to the stationary monetary competitive equilibrium of Chapter II. Under the conditions that  $\beta_1 < 1 + \alpha$ ,  $\beta_2 \le 1 + \alpha$ , and  $1 + \alpha \le \beta_2 \delta_2 f'_2(\overline{L}_2)/\delta_1 f'_1(\overline{L}_1)$  there exist a nonstochastic steady state. Steady state, whenever exists, satisfies (3.25)-(3.36) with  $z_{it} = 0$  for all t and i = 1, 2, all prices  $(p_t, w_t, p_t^e)$  growing at rate  $\alpha$  and all real variables  $(L_{it}, f_{it}, c_{it}, q_{it}, n_i, X_{it})$ , for i = 1, 2, being constant over time. Under the price regulation  $(p_t^e/p_t)$  is set by the regulator, and the total amount of emissions  $E_t$  results from market interactions. Conversely, under quantity regulation,  $E_t$  is fixed by the regulator at  $\overline{X}_t$  and  $p_t^e$  is determined in the permits market.

Remark 7 In the absence of policy response, the labor demand of the cash-constrained

agent declines with an increase in  $\theta$ .

**Proof.** In the non-stochastic steady state, equation (3.25) becomes:

$$\frac{\beta_2 \delta_{2t} f'_{2t}}{(1+\alpha)\theta + \beta_2 (1-\theta)} = \delta_{1t} f'_{1t}$$
(3.37)

Taking the logarithm of both sides of the equality and differentiating w.r.t.  $\theta$ , we get:

$$\frac{\frac{\partial \delta_{2}}{\partial (p_{t}^{e}/p_{t})} \frac{\partial (p_{t}^{e}/p_{t})}{\partial L_{2t}} \frac{\partial L_{2t}}{\partial \theta}}{\delta_{2t}} + \frac{f_{2}^{\prime\prime}}{f_{2}^{\prime}} \frac{\partial L_{2t}}{\partial \theta} = -\frac{N_{2}}{N_{1}} \frac{f_{1}^{\prime\prime}}{f_{1}^{\prime}} \frac{\partial L_{2t}}{\partial \theta} + \frac{\frac{\partial \delta_{1}}{\partial L_{2t}} \frac{\partial (p_{t}^{e}/p_{t})}{\partial L_{2t}} \frac{\partial (p_{t}^{e}/p_{t})}{\partial \theta}}{\delta_{1t}} + \frac{1 + \alpha - \beta_{2}}{(1 + \alpha)\theta + \beta_{2}(1 - \theta)}$$
(3.38)

Substituting in for  $\frac{\partial \delta_i}{\partial (p_t^e/p_t)}$  from equation (2.32) and simplifying we get the following expressions for the response of  $L_{2t}$  to  $\theta$  under price and quantity regulations, respectively:

$$\frac{\partial L_{2t}^{P}}{\partial \theta} = \frac{\frac{1+\alpha-\beta_{2}}{(1+\alpha)\theta+\beta_{2}(1-\theta)}}{\frac{f_{2}''}{f_{2}'} + \frac{N_{2}}{N_{1}}\frac{f_{1}''}{f_{1}'}} < 0$$
(3.39)

$$\frac{\partial L_{2t}^{Q}}{\partial \theta} = \frac{\frac{1+\alpha-\beta_{2}}{(1+\alpha)\theta+\beta_{2}(1-\theta)}}{\frac{f_{2}^{\prime\prime}}{f_{2}^{\prime\prime}} + \frac{N_{2}}{N_{1}}\frac{f_{1}^{\prime\prime}}{f_{1}^{\prime\prime}} + \frac{\partial(p_{t}^{e}/p_{t})}{\partial L_{2t}}\left(\frac{(1-s_{1t})\gamma_{1}}{\delta_{1}} - \frac{(1-s_{2t})\gamma_{2}}{\delta_{2}}\right)} < 0$$
(3.40)

where superscripts P and Q denote price and quantity regulation, respectively. In

the expressions above, the nominators are positive, as indicated by the conditions for the existence of stationary equilibrium from Proposition 1 from Chapter II. The denominator in (3.39) is negative due to assumptions about the production technology. The denominator in (3.40) is also negative because the derivative of the real permit price with respect to  $L_{2t}$  is positive, and the expression multiplying the derivative within parenthesis is negative. We show that permit price increases with  $L_{2t}$  in section (3.5.a.). The term in parenthesis is negative due to the concavity of the abatement technology. This is proved in Corollary 1.1 in Chapter II. In the following, we conduct numerical experiments to illustrate how equilibrium and the optimal regulatory policy change with respect to the scale of the cash constraint. For the numerical exercises in this chapter we use the parametrization outlined in Table 3.1. It is important to note that the values in Table 3.1 are not calibrated to match any economic data. The primary aim of these numerical experiments is to offer insights into the model dynamics. Additionally, we simplify the analysis with the following assumption for numerical exercises throughout the paper: Only type 2 agent is pollutant,  $\gamma_1 = 0$ ; only type 2 agents face productivity shocks,  $z_{1t} = 0$  for all t; utility in consumption is logarithmic; collected fines are not returned to agents,  $T_1 = T_2 = 0$ ; Type 2 agent is not endowed with labor,  $\overline{L}_2 = 0$ .

Notation	Description	Value
$\gamma_1$	0	Pollution rate of type 1 agent
$\gamma_2$	0.6	Pollution rate of type 2 agent
$\beta_2$	0.9	Discount factor of type 2 agent
$\lambda_1$	0.2	Output elasticity of type 1 agent
$\lambda_2$	0.6	Output elasticity of type 2 agent
B <sup>′</sup>	0.1	Disutility from pollution
$\alpha_1$	0.1	Share of money stock allocated to type 1 agent
$\alpha_2$	0	Share of money stock allocated to type 2 agent
$\overline{L}_1, \overline{L}_2$	10, 0	Labor endowments
$A_1$	1	Total factor productivity of type 1 agent
$A_2$	2.5	Total factor productivity of type 1 agent
$\varepsilon_1 = \varepsilon_2 = \varepsilon$	2	Parameter setting the curvature of the abatement technology
κ	1/40	Constant in the abatement technology
η	0.9	Persistence of the productivity shock
$N_1, N_2$	(0.8, 0.2)	Distribution of the population
$\theta$	(0.1, 0.18)	Parameter defining tightness of the cash constraint
η	0.9	Persistence of the productivity shock
$\sigma_{epsilon}$	0.5	Standard deviation of the productivity shock

Table 3.1. Parameter Set for the Numerical Exercises in Chapter III

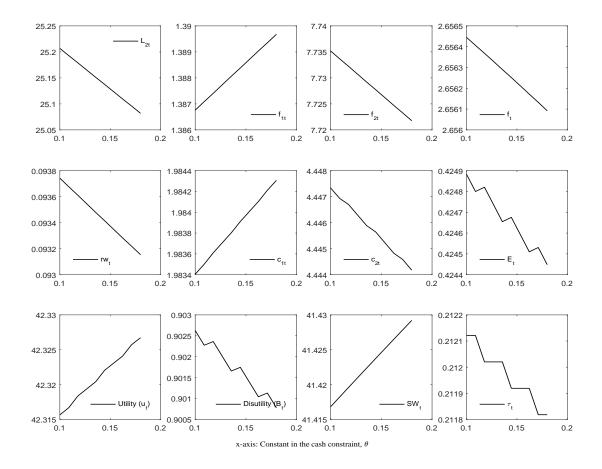
As in Chapter II, this parametrization implies a case where type 1 agent consumes less than the type 2 agent, i.e.  $c_{1t} < c_{2t}$ . Therefore, if money growth occurs through transfers to type 1 agent, as is the case in our framework, stricter cash constraint favors the consumption of the type 1 agent. In the model, the money stock is equal to the money transfers of the type 2 agent for the advance payment of labor expenses. A tighter cash constraint, i.e., higher  $\theta$ , results in an increased money base, which translates to higher money transfers to type 1 agent.

Figure 3.3 illustrates the response of the non-stochastic steady state to  $\theta$  under price policy given the parametrization in Table 3.1. The monetary policy is fixed, and the underlying tax rate for each  $\theta$  is the one that maximizes the expected social welfare. Social welfare is defined as the discounted lifetime utility from consumption and disutility from pollution. The maximization procedure is based on a grid search over  $\tau$ ; therefore, policy updating is discrete.<sup>4</sup> Notice that, without the policy response, as the required rate of advance payment in the labor market increases, the labor demand of the type 2 agent decreases, leading to a shift in production away from the cash-constrained more productive agent. This results in a decrease in total output and emissions. Under this parametrization, the consumption of the type 2 agent declines, while that of the type 1 agent increases. A higher amount of money transfers compensates for the decline in the labor income of type 1 agents, leading to a higher consumption for type 1 agents. As explained earlier, money transfers to type 1 agents are proportional to  $\theta$ . Overall, utility from consumption improves as the consumption allocation shifts in favor of the agent that consumes less, reducing consumption inequality.

As a result, social welfare improves due to both higher utility of consumption and lower disutility from emissions. From the social planner's perspective, the decline in emissions implies a reduction in the benefits of implementing a stricter regulatory policy. At the same time, higher utility from consumption implies a decline in the cost of implementing stricter regulatory policy. Both factors prompt the regulator to adopt a looser tax policy. As the tax rate adjusts downward, consumption and the utility from consumption adjust upward.

<sup>&</sup>lt;sup>4</sup>We maximize quadratic approximation to expected welfare. The details of this approach are provided in Section 3.7.a.

**Figure 3.3.** Non-Stochastic Steady State Under Optimal Price Policy  $(p_t^e/p_t)$ 



#### 3.5.a. Price in the Asset Market

In this section, we derive the price of emission permits and show that the demand for emission permits is decreasing in the price of permits. For this purpose, we explicitly define the functional form of the technology that converts output into pollution control units as:

$$s_i(n_{it}) = 1 + \kappa \frac{n_{it}^{1-\varepsilon_i}}{1-\varepsilon_i} \quad \text{for i=1,2}$$
(3.41)

Using first-order conditions for  $n_i$  (eq. 3.27) together with equation (3.41), we get the following expression for  $n_{it}$ :

$$n_{it} = \left(\frac{p_t^e}{p_t}\gamma_i\kappa\right)^{(1/\varepsilon_i)} \quad \text{for } i = 1,2$$
(3.42)

Equilibrium in the asset market requires that equation (3.29) holds. Substituting for  $s_i$  in the asset market equilibrium condition we get:

$$N_{2}(\kappa\gamma_{2})^{\frac{1}{\epsilon_{2}}} \frac{\left(\frac{p_{t}^{e}}{p_{t}}\right)^{\frac{1-\epsilon_{2}}{\epsilon_{2}}}}{\epsilon_{2}-1} e^{z_{2}} f_{2t} + N_{1}(\kappa\gamma_{1})^{\frac{1}{\epsilon_{1}}} \frac{\left(\frac{p_{t}^{e}}{p_{t}}\right)^{\frac{1-\epsilon_{1}}{\epsilon_{1}}}}{\epsilon_{1}-1} e^{z_{1}} f_{1t} = \overline{X}_{t}$$
(3.43)

We make a further simplification and assume that  $\varepsilon_1 = \varepsilon_2 = \varepsilon$ . Then we get:

$$\frac{p_t^e}{p_t} = \left(\frac{\overline{X}_t \left(\varepsilon - 1\right)}{\left(N_2 \gamma_2^{\frac{1}{\varepsilon}} e^{z_2} f_{2t} + N_1 \gamma_1^{\frac{1}{\varepsilon}} e^{z_1} f_{1t}\right) \kappa^{\frac{1}{\varepsilon}}}\right)^{\frac{\varepsilon}{1-\varepsilon}}$$
(3.44)

**Remark 8** Assuming that  $s_i$  is increasing and concave in  $n_i$ , i.e.  $\varepsilon_i > 1$ , the market for permits exits. Moreover, i) is a positive price for which demand equals supply, and the ii) demand curve for permits is downward sloping.

The proof of the statement in (i) directly follows from equation (3.44). In order

to prove (ii), we differentiate the demand for permits w.r.t. the real price of permits:

$$\frac{\partial \sum_{i} N_{i} E_{it}}{\partial \left(p_{t}^{e}/p_{t}\right)} = -\frac{\varepsilon - 1}{\varepsilon} \kappa^{2} \left(\gamma_{1}^{\frac{1}{\varepsilon}+2} e^{z_{1t}} f_{1t} + \frac{N_{2}}{N_{1}} \gamma_{2}^{\frac{1}{\varepsilon}+2} e^{z_{2t}} f_{2t}\right)$$
(3.45)

$$+ \frac{p_t}{p_t^e} \left( \gamma_1^{\frac{1}{e}} e^{z_1} f_{1t}' - \gamma_2^{\frac{1}{e}} e^{z_2} f_{2t}' \right) \frac{\partial L_{1t}}{\partial \left( p_t^e / p_t \right)}$$
(3.46)

The first component of the derivative is always negative given  $\varepsilon > 1$ . This part represents the response of  $n_i$  to a higher price. In response to a higher price, agents reserve more of output for pollution control and demand for permits decline. The second part defines the change in demand due to the shift in production between agents. Given that  $\varepsilon > 1$ , this part is also negative. This point is demonstrated using the results from Corollary 1.1 and Proposition 2 from Chapter II. Recall from Corollary 1.1 that when  $\varepsilon > 1$ , the labor of the more pollutant type declines in response to a higher price of pollution. Furthermore, recall from Proposition 2.1, in the presence of a pollution externality, the production of the more pollutant type is lower than in the case where total output in the economy is maximized. Using these results, if agent 1 is the more pollutant type,  $(\gamma_1^{\frac{1}{\epsilon}} f'_{1t} > \gamma_2^{\frac{1}{\epsilon}} f'_{2t}) > 0$  and  $\frac{\partial L_{1t}}{\partial (p_t^{\epsilon}/p_t)} > 0$ . In contrast, if agent 2 is the more pollutant type,  $(\gamma_1^{\frac{1}{\epsilon}} f'_{1t} < \gamma_2^{\frac{1}{\epsilon}} f'_{2t}) > 0$  and  $\frac{\partial L_{1t}}{\partial (p_t^{\epsilon}/p_t)} > 0$ . In both cases, the second component of the derivative is negative.

**Remark 9** Given that the supply of emissions is inelastic to price and the demand curve is negatively sloped, the relative price of permits decreases with an increase in the cap on emissions.

In the next section, we log-linearize the stochastic solution to study the system's dynamics under productivity shocks.

#### 3.6. Log-linear Version of the Model

In the presence of productivity shocks, real variables no longer remain constant across consecutive periods. The main equation governing labor allocation is given by 3.25. Following this nonlinear equation for dynamics can be challenging. To facilitate understanding, we log-linearize the system using the methodology outlined in the Appendix. Furthermore, we use the parameter set detailed in Table 3.1, maintain the simplifying assumptions introduced in Section 3.5., and assume the following functional forms:

$$u(c_{it}) = ln(c_{it}) \tag{3.48}$$

$$f_{it} = A_i (L_{it} + \overline{L_i})^{\lambda_i} \text{ for } i = 1,2$$
(3.49)

$$s_i(n_{it}) = 1 + \kappa \frac{n_{it}^{1-\varepsilon_i}}{1-\varepsilon_i}$$
(3.50)

Log-linearizing the system of equations (3.25) to (3.36), the main optimality condition given in (3.25) and its components become:

$$\widehat{w}_{t} - \widehat{w}_{t+1} + \widehat{\delta}_{2t+1} z_{2t+1} + \widehat{f}_{2t+1}' - \frac{\theta}{\theta + \beta_{2}(1-\theta)} (\widehat{c}_{2t+1} - \widehat{c}_{2t}) - \frac{\beta_{2}(1-\theta)}{\theta + \beta_{2}(1-\theta)} (\widehat{p}_{t} - \widehat{p}_{t+1}) - \widehat{f}_{1t}' - z_{1t} = 0$$
(3.51)

$$\widehat{w}_t - \widehat{w}_{t+1} = \widehat{l}_{2t+1} - \widehat{l}_{2t}$$
(3.52)

$$\widehat{w}_t - \widehat{p}_t = \widehat{f}'_{1t} + z_{1t} \tag{3.53}$$

$$\widehat{p}_{t+1} - \widehat{p}_t = \widehat{w}_{t+1} - \widehat{w}_t + \widehat{f}'_{1t} + z_{1t} - \widehat{f}'_{1t+1} - z_{1t+1}$$
(3.54)

$$\frac{L_{1s} + \bar{L}_1}{L_{1s}} \widehat{L_{1t} + \bar{L}_1} = \widehat{l}_{2t}$$
(3.55)

$$\widehat{f}_{1t} = \lambda_1 \widehat{L_{1t} + \overline{L}_{1s}}$$
(3.56)

$$\widehat{f}_{1t}^{\prime} = (\lambda_1 - 1)\widehat{L_{1t}} + \overline{\overline{L}}_{1s}$$
(3.57)

$$\widehat{f}_{2t} = \lambda_2 \widehat{l}_{2t} \tag{3.58}$$

$$\hat{f}_{2t}^{\prime} = (\lambda_2 - 1)\,\hat{l}_{2t} \tag{3.59}$$

$$\widehat{u}_{2t}' = -\widehat{c}_{2t} \tag{3.60}$$

$$\widehat{c}_{1t} = \frac{f_{1s}}{c_{1s}}(\widehat{f}_{1t}) + \frac{N_2}{N_1}\omega L_{2s}\frac{1 + \alpha_2 + \theta\,\alpha_1}{(1 + \alpha_2)c_{2s}}\left(\widehat{w}_t - \widehat{p}_t + \widehat{l}_{2t}\right)$$
(3.61)

$$\widehat{c}_{2t} = \frac{\delta_{2s} f_{2s}}{c_{2s}} (\widehat{\delta}_{2t} + z_{2t} + \widehat{f}_{2t}) - \omega L_{2s} \frac{1 + \alpha_2 + \theta \alpha_1}{(1 + \alpha_2)c_{2s}} \left( \widehat{w}_t - \widehat{p}_t + \widehat{l}_{2t} \right)$$
(3.62)

These equations represent the system's deviation from the non-stochastic steady state. Variables denoted with a hat represent log deviations, and the subscript *s* denotes the steady state. Since the real price per emission is fixed, the terms  $\hat{\delta}_{2t+1}$  in the above equations are null under price regulation. However, under quantity regulation, the price of permits responds to output. Therefore  $\delta_{2t+1}$  responds to the deviation of  $(p_t^e/p_t)$  from its steady-state value. Hence, we must incorporate the following equations into the log-linearized system under quantity regulation.

$$\hat{p}_t^e - \hat{p}_t = \psi_2(z_{2t} + f_{2t})$$
(3.63)

$$\widehat{n}_{2t} = \frac{1}{\varepsilon} (\widehat{p}_t^e - \widehat{p}_t) \tag{3.64}$$

$$\widehat{s}_{2t} = \frac{\kappa n_{2s}^{1-\varepsilon}}{s_{2s}} \widehat{n}_{2t} \tag{3.65}$$

$$\widehat{\delta}_{2t} = -\frac{n_{2s}}{\delta_{2s}}\widehat{n}_{2t} - \frac{p_s^e}{p_s}\frac{(1-s_{2s})\gamma_2}{\delta_{2s}}(\widehat{p}_t^e - \widehat{p}_t) + \frac{p_s^e}{p_s}\frac{s_{2s}\gamma_2}{\delta_{2s}}\widehat{s}_{2t}$$
(3.66)

Substituting in  $\hat{\delta}_{2t}$  for  $\hat{n}_{2t}, \hat{s}_{2t}, s_{2s}$  from equations (3.64), (3.65), (3.41), we get,

$$\widehat{\delta}_{2t} = \zeta_2 (\widehat{p}_t^e - \widehat{p}_t), \qquad (3.67)$$

where,  $\psi_2 = \frac{\varepsilon}{\varepsilon - 1}$  and  $\zeta_2 = \frac{n_{2s}}{\delta_{2s}(1 - \varepsilon)}$ .

**Log-linearized Dynamics Under Price Regulation** Equations (3.51)-(3.62), with  $\hat{\delta}_{2t} = 0$  for all *t*, are enough to define the system under price control. Substituting in (3.51) from (3.52)-(3.62) we get :

$$\Omega_1 z_{2t} + \Omega_2 z_{2t+1} + \Omega_3 \widehat{l}_{2t} + \Omega_4 \widehat{l}_{2t+1} = 0, \qquad (3.68)$$

where,

$$\Omega_{1} = \frac{\theta}{\beta_{2}(1-\theta)+\theta} \frac{f_{2s}\delta_{2s}}{c_{2s}}$$

$$\Omega_{2} = 1-\Omega_{1}$$

$$\Omega_{3} = \Omega_{1} \left(\lambda_{2} - \left(1 - \frac{L_{1s}}{L_{1s} + \overline{L}_{1}}(1-\lambda_{1})\right)\right)$$

$$\Omega_{4} = (1-\Omega_{1}) \left(\lambda_{2} - \left(1 - \frac{L_{1s}}{L_{1s} + \overline{L}_{1}}(1-\lambda_{1})\right)\right)$$
(3.69)

**Log-linearized Dynamics Under Quantity Regulation** Under quantity regulation, equations (3.51)-(3.67) define the system. Substituting in (3.51) from (3.52)-(3.67) we get :

$$\Gamma_1 z_{2t} + \Gamma_2 z_{2t+1} + \Gamma_3 l_{2t} + \Gamma_4 l_{2t+1} = 0, \qquad (3.70)$$

with,

$$\Gamma_{1} = (1 + \psi_{2}\zeta_{2})\Omega_{1}$$

$$\Gamma_{2} = (1 + \psi_{2}\zeta_{2})(1 - \Omega_{1})$$

$$\Gamma_{3} = \Omega_{3} + (\lambda_{2}\psi_{2}\zeta_{2})(\Omega_{1})$$

$$\Gamma_{4} = \Omega_{4} + (\lambda_{2}\psi_{2}\zeta_{2})(1 - \Omega_{1})$$
(3.71)

where  $\psi_2 = \frac{\varepsilon}{\varepsilon - 1}$  is the response of permit prices to the output of type 2 agent and  $\zeta_2$  is the response of  $\delta_2$  to permit prices.

**Stability of the System** The non-stochastic system exists if the system is stable. The following proposition states the condition under which the stability of the system is ensured.

## **Proposition 4** The system is not stable under full cash-in-advance constraint.

**Proof.** Under price and quantity control frameworks stability requires that,  $\left|-\frac{\Omega_3}{\Omega_4}\right| < 1$  and  $\left|-\frac{\Gamma_3}{\Gamma_4}\right| < 1$ , respectively. Using the definitions given in equations (3.69) and (3.71), these conditions simplify as  $\left|-\frac{\Omega_1}{(1-\Omega_1)}\right| < 1$ . Using equations (3.69),  $\Omega_1 > 0$ . Therefore,  $\left|-\frac{\Omega_1}{(1-\Omega_1)}\right| = -\frac{\Omega_1}{(1-\Omega_1)}$  if  $\Omega_1 > 1$ ;  $\left|-\frac{\Omega_1}{(1-\Omega_1)}\right| = \frac{\Omega_1}{(1-\Omega_1)}$  if  $\Omega_1 < 1$ . Consider the first case where  $\Omega_1 > 1$ . Stability requires:

$$-\frac{\Omega_1}{(1-\Omega_1)} < 1 \tag{3.72}$$

$$\Omega_1 < \Omega_1 - 1 \tag{3.73}$$

$$0 < -1$$
 (3.74)

This is a contradiction. Consider the second case where  $\Omega_1 < 1$ . Now we have:

$$\frac{\Omega_1}{(1-\Omega_1)} < 1 \tag{3.75}$$

$$\Omega_1 < \frac{1}{2} \tag{3.76}$$

$$\frac{\theta}{\beta_2(1-\theta)+\theta}\frac{f_{2s}\delta_2}{c_{2s}} < \frac{1}{2}$$
(3.77)

Since consumption must be positive,  $\frac{f_{2s}\delta_2}{c_{2s}} > 1$ , we must have:

$$\frac{\theta}{\beta_2(1-\theta)+\theta} < \frac{1}{2} \tag{3.78}$$

$$\theta < \frac{\beta_2}{1+\beta_2} \tag{3.79}$$

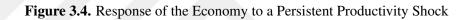
The proof is complete.

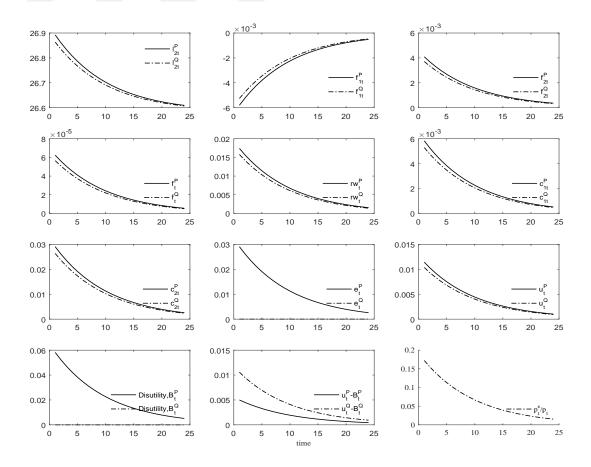
**Impact of a Persistent Productivity Shock** In the following, we explore the dynamics of the system assuming that the type 2 agent is subject to a persistent productivity shock given as,

$$z_{2t+1} = \eta z_{2t} + v_t \tag{3.80}$$

where  $v_t$  is a random shock with mean zero and standard deviation  $\sigma_v$ . Figure 3.4 demonstrates the response of the system to a positive productivity shock under optimal tax policy and quantity policy, where the cap is the expected emissions under optimal tax policy. We use grid search to identify the optimal tax rate by maximizing the expected social welfare, which is defined as the sum of discounted utility from consumption and disutility from pollution for all agents over all periods.<sup>5</sup> Then, we compute the mean of the expected emissions over time under optimal tax policy. Using this value as the cap on emissions, we compute the non-stochastic steady state and response to a productivity shock under quantity policy. This way, the two policies we consider share the same non-stochastic steady state. The figure shows that volatility is lower under quantity policy. The response of variables to a positive productivity shock is less pronounced under quantity regulation, as the increase in output stimulates a price increase in the permits market, requiring more output to be reserved for pollution control. The cost of production increases, weakening the response of labor to productivity shock.

<sup>&</sup>lt;sup>5</sup>Expected social welfare is computed numerically by drawing 10000 values of  $v_1$  from a uniform distribution.





Note: variables are in log deviations from steady state. Superscript P and Q represent price and quantity controls, respectively.

**Impact of Independent Productivity Shock** In this part, we compute the variation in labor and consumption under price and quantity regulations, assuming productivity shocks are independently and identically distributed (i.i.d.) ( $\eta = 0$ ). Under this assumption, we derive the second-order moments of the main macro variables analytically. Using equation (3.68) we can compute the mean and the variance of  $\hat{l}_{2t}$ . Taking expectations of the equation (3.68) we get:

$$\Omega_{3}E\left(\widehat{l}_{2t}\right) + \Omega_{4}E\left(\widehat{l}_{2t+1}\right) = 0$$
(3.81)

Given that  $\Omega_3 \neq \Omega_4 \neq 0$ , above equality holds only if  $E(\hat{l}_{2t}) = E(\hat{l}_{2t+1}) = 0$ . Then,  $E(L_{2t}) = L_{2t}$ . The productivity shocks are i.i.d, therefore  $\hat{l}_{2t}$  is not correlated with any of the shocks that occur after time *t*, and the shocks are not correlated between periods, i.e.,  $E(z_{2j}, z_{2k}) = 0$  for  $j \neq k$ . Using the equation (3.68) and assuming i.i.d shocks, we can compute the variation in  $\hat{l}_{2t}$ . Multiplying equation (3.68) separately with  $z_{it}, z_{it+1}, \hat{l}_{2t}, \hat{l}_{2t+1}$  and taking expectations we get:

$$\Omega_1 \sigma_{z2}^2 + \Omega_3 E[\hat{l}_{2t}, z_{2t}] + \Omega_4 E[\hat{l}_{2t+1}, z_{2t}] = 0 \quad (3.82)$$

$$\Omega_2 \sigma_{z2}^2 + \Omega_4 E(\hat{l}_{2t}, z_{2t}) = 0 \quad (3.83)$$

$$\Omega_1 E[\hat{l}_{2t}, z_{2t}] + \Omega_3 Var[\hat{l}_{2t}] + \Omega_4 E[\hat{l}_{2t+1}, \hat{l}_{2t}] = 0 \quad (3.84)$$

$$\Omega_1 E[\hat{l}_{2t+1}, z_{2t}] + \Omega_2 E[\hat{l}_{2t}, z_{2t}] + \Omega_3 E[\hat{l}_{2t+1}, \hat{l}_{2t}] + \Omega_4 Var[\hat{l}_{2t}] = 0 \quad (3.85)$$

Solving these equations together, and substituting in from equation set (3.69), we obtain the expression for the variance of  $\hat{l}_{2t}$  and the covariance between  $\hat{l}_{2t}$  and  $z_{2t}$  as:

$$Var[\hat{l}_{2t}^{P}] = \frac{1}{(\lambda_{2} - \tilde{\lambda}_{1})^{2}} \sigma_{z2}^{2}, \quad E(\hat{l}_{2t}^{P}, z_{2t}) = -\frac{1}{(\lambda_{2} - \tilde{\lambda}_{1})} \sigma_{z2}^{2}$$
(3.86)

where *P* denotes price control and  $\tilde{\lambda}_1 = 1 - \frac{L_{1s}}{L_{1s} + \overline{L_1}} (1 - \lambda_1) > 1$ . Following the same steps, under quantity regulation the variance of labor of the type 2 agent and the

covariance between  $\hat{l}_{2t}$  and  $z_{2t}$  are given by:

$$Var[\hat{l}_{2t}^{Q}] = \left(\frac{1}{\lambda_{2} - \frac{\tilde{\lambda}_{1}}{1 + \psi_{2}\zeta_{2}}}\right)^{2} \sigma_{z2}^{2}, \quad E(\hat{l}_{2t}^{Q}, z_{2t}) = -\frac{1}{\lambda_{2} - \frac{\tilde{\lambda}_{1}}{1 + \psi_{2}\zeta_{2}}} \sigma_{z2}^{2}$$
(3.87)

Using the log-linearized equations for  $c_{2t}$  and the variance and covariance of  $\hat{l}_{2t}$ and  $z_{2t}$ , we can compute the variance of consumption as:

$$Var(\hat{c}_{2t}^{P}) = \sigma_{z_{2t}}^{2} \left(\frac{\tilde{\lambda}_{1}}{\tilde{\lambda}_{1} - \lambda_{2}}\right)^{2} \quad Var(\hat{c}_{2t}^{Q}) = \sigma_{z_{2t}}^{2} \left(\frac{\tilde{\lambda}_{1}}{\frac{\tilde{\lambda}_{1}}{1 + \psi_{2}\zeta_{2}} - \lambda_{2}}\right)^{2}$$
(3.88)

To obtain the expressions for the variance of consumption, we first substitute for  $\hat{\delta}_{t2}$ ,  $\hat{f}_{2t}$  and  $\hat{f'}_{1t}$  in the equation for  $\hat{c}_{t2}$ . Then, we square both sides and take expectations. Once we substitute in for  $Var[\hat{l}_{2t}]$ ,  $Cov[\hat{l}_{2t}, z_{2t}]$ , and use the expression for  $c_{2t}$  from the non-stochastic steady state we get the variance of consumption.

Looking at equations (3.86) and (3.87), the difference between second-order moments is the term  $\psi_2 \zeta_2$ , which determines the response of  $\delta_2$  to a change in output. It is multiple of the impact of the response of permit prices to output ( $\psi_2$ ), and the response of  $\delta_2$  to change in permit prices ( $\zeta_2$ ). Given that the abatement technology is concave in  $n_2$ ,  $\psi_2 > 0$  (eq. 3.63) and  $\zeta_2 < 0$  (eq. 3.67). Furthermore, notice that the covariance between  $\hat{l}_{2t}$  and  $z_{2t}$  is positive only if  $\psi_2 \zeta_2 < -1$ . Otherwise, the cost channel due to permit price is so dominant that labor demand by type 2 agent declines in response to a positive productivity shock.

**Remark 10** Assuming that  $-1 < \psi_2 \zeta_2 = -\frac{\varepsilon}{(\varepsilon-)^2} \frac{n_{2s}}{\delta_{2s}} < 0$ , the variation in labor is lower under quantity regulation compared to price regulation. Along with the variation in labor, variation in consumption is also lower under quantity regulation.

The above remark is easily verified using the definitions of variance and covariances given in equations (3.86) and (3.87).

In the following, we examine the association between the degree of nominal rigid-

ity and the variation in variables. We investigate how an increase in nominal rigidity affects the variation under price and control regulation.

**Remark 11** The variance of  $\hat{l}_{2t}$  increases with  $\theta$  under both regulations. However, given that the impact of the permits market channel on net revenues decreases with  $\theta$  ( $\psi_2 \zeta_2$  is increasing in  $\theta$ ), the variance under quantity regulation exhibits a less pronounced increase compared to the variance under price regulation with  $\theta$ .

The variance of  $\hat{l}_{2t}$  is affected by  $\theta$  through its impact on the equilibrium labor allocation. Note that the variance of labor (equations 3.86 and 3.87) is decreasing in  $\tilde{\lambda}_1$ , which in turn is decreasing in  $L_{1s}$ . Furthermore,  $L_{1s}$  increases with  $\theta$  (see Remark 7). Additionally, under quantity regulation,  $\psi_2 \zeta_2$  increases with  $\theta$ . To see this, recall that higher  $\theta$  reduces labor demand of the type 2 agent. Under quantity regulation, this reduction in labor demand shifts the demand for permits downward, leading to a decrease in the price of permits. In the following, we will demonstrate that  $\psi_2 \zeta_2$  is decreasing in the price of permits.

$$\frac{\partial \psi_2 \zeta_2}{\partial (p_t^e/p_t)} = -\frac{\varepsilon}{\varepsilon - 1} \frac{\partial \frac{n_{2s}}{\delta 2s}}{\partial (p_t^e/p_t)}$$
(3.89)

$$= -\frac{\varepsilon}{\varepsilon - 1} \left( \frac{1}{\delta_{2s}} \frac{\partial n_{2s}}{\partial (p_t^e/p_t)} - \frac{1}{\delta_{2s}^2} \frac{\delta_{2s}}{\partial (p_t^e/p_t)} \right)$$
(3.90)

$$= -\frac{\varepsilon}{\varepsilon - 1} \left( \frac{1}{\delta_{2s}} \frac{\partial n_{2s}}{\partial (p_t^e/p_t)} + \frac{1}{\delta_{2s}^2} (1 - s_2) \gamma_2 \right) < 0$$
(3.91)

Looking at the derivations above, in the last line, the first fraction within the parenthesis is positive since  $n_{2s}$  increases with  $p_t^e/p_t$ . The second fraction in the parenthesis follows from equation (2.32) and it is greater than zero. Hence derivative of  $\psi_2 \zeta_2$ with respect to  $p_t^e/p_t$  is negative. Eventually, the fact that  $\psi_2 \zeta_2$  increases with  $\theta$ , is a factor reducing variance. In other words, the erosion in revenues in response to higher permit price is lower under higher  $\theta$ . Therefore, the variance under quantity regulation increases less with  $\theta$ .

#### 3.7. Social Welfare Under Alternative Policies

In this part, we introduce a framework for comparing the expected social welfare under alternative regulatory policies practiced at their optimal. Given the non linearity of the model this comparison is not straightforward. Therefore, initially, we conceptualize the comparison between the expected social welfare under optimal quantity and optimal price policy, but we leave implementation of this for further research and focus on a more feasible comparison of alternative policies around the same steady state.

Below, we define social welfare and the expected welfare under the optimal price and quantity policies. Social welfare is the sum of discounted utility from consumption and the disutility from pollution for all agents over all periods:

$$SW = \sum_{t=0}^{\infty} \sum_{i} \beta_i^t (u_i(c_{it}) - B(E_t))$$

Following Kelly (2005), the expected welfare under optimal quantity policy  $\widehat{E}_t$  is given by:

$$\widehat{v} = \max_{E_t} E[SW(z_{2t}, \widehat{E}_t)]$$

The expected welfare under optimal price policy is given by:

$$\tilde{v} = \max_{\frac{\tilde{p}_t^e}{p_t}} E[SW(z_{2t}, \tilde{E}(z_{2t}, \frac{\tilde{p}_t^e}{p_t}))]$$

The expected welfare gain under quantity regulation over price regulation is,  $\Delta \equiv \hat{v} - \tilde{v}$ . The difficulty here is that numerical computations with nonlinear social welfare functions are complex (sometimes not feasible). Therefore, for simplicity we use the quadratic approximation to social welfare as defined in the following section.

#### 3.7.a. Quadratic Approximation to Social Welfare Function

Woodford (2002) advocates the use of quadratic approximation (second-order Taylor series) to the objective function. Accordingly, given utility function  $U_t = u(x, \zeta)$  and  $E[\zeta] = 0$ , the expected value of the quadratic approximation is given by:

$$E[U] = \overline{U} + U_x E[\widetilde{x}] + \frac{1}{2} U_{xx} var[\widetilde{x}] + \frac{1}{2} U_{\zeta\zeta} var[\zeta] + U_{x\zeta} cov[\zeta, \widetilde{x}] + \mathcal{O}$$

where variables with a tilde sign above represent non-stochastic steady state values,  $\overline{U} = U(\overline{x}, \zeta)$ , and  $\overline{x} = x - \overline{x}$  is deviation from steady state. All partial derivatives are evaluated at  $(\overline{x}, 0)$ .  $\mathcal{O}$  represent terms that are of  $3^{rd}$  degree or higher. We also follow this method to compute a quadratic approximation to social welfare. This approach simplifies numerical computations, with quadratic approximation to the objective function and linear approximation to the structural equations. Quadratic approximation to social welfare under optimal quantity and price policies is given by:

$$E[SW^{k}(L_{2t}^{k}, z_{2t})] = S\bar{W}^{k} + \frac{1}{2}SW_{L_{2},L_{2}}^{k}E[(L_{2t}^{k} - \bar{L}_{2})^{2}] + \frac{1}{2}SW_{z_{2},z_{2}}^{k}\sigma_{z_{2}}^{2} + SW_{L_{2},z_{2}}^{k}E[(L_{2t}^{k} - \bar{L}_{2})z_{2}] \quad k=Q, P$$
(3.92)

where Q and P represent quantity and price policies respectively.  $SW^k$  is the social welfare under non-stochastic steady state,  $SW^k_{L_2,L_2}$ ,  $SW^k_{z_2,z_2}$  and  $SW^k_{L_2,z_2}$  are partial derivatives evaluated at the non-stochastic steady state. Steady states under optimal quantity and optimal price policy would be different. The comparison of the optimal price and quantity regulations requires carrying out a numerical optimization routine under each regulatory framework. This has to be done consistently, which is a complex task. Therefore, we leave it for further research. Instead, we make a partial comparison focusing on the second-order terms in the quadratic approximation to the social welfare function.

Kelly (2005) shows that to make the point that quantity regulation is better than

price regulation, i.e.  $E[SW^Q(\widehat{E}_t)] > E[SW^P(\widetilde{E})(z_{2t}, \frac{p_t^e}{p_t}))]$ , it is enough to show that:

$$E[SW^{Q}(\overline{E})] > E[SW^{P}(\tilde{E}(z_{2t}, \frac{\tilde{p}_{t}^{e}}{p_{t}}))]$$
(3.93)

where  $\overline{E}$  is equal to the expected emissions under optimal price policy, i.e.  $\overline{E}_t = E[\tilde{E}_t(z_{2t}, \frac{\tilde{p}_t^e}{p_t})]$ . This follows from the reasoning that if  $\hat{E}$  is optimal, then it must be better compared to any other quantity, including  $\overline{E}$ . Then, to make the point that quantity regulation is better than price regulation, it is enough to show that inequality (3.93) holds.

In this context, we conduct a numerical exercise to compute social welfare under the optimal price policy and the quantity policy, where quantity is not the optimal quantity that maximizes the expected social welfare but it is expected emissions under the optimal price policy. In this case, social welfare under non-stochastic steady state, denoted as  $S\bar{W}^k$ , is identical for quantity and price policy. The partial derivatives,  $SW_{L_2,L_2}^k$ ,  $SW_{z_2,z_2}^k$  and  $SW_{L_2,z_2}^k$ , are different across regulations but they are evaluated at the same steady state. Using this quadratic approximation and the variance and the covariances given in the previous sections, namely  $Var[\hat{l}_{2t}^k]$ ,  $Cov[\hat{l}_{2t}^k, z_{2t}]$ , we can compute social welfare under each policy according to equation 3.92. The derivation of the partial derivative functions is provided in the Appendix.

Figure 3.5 illustrates how the expected social welfare and the variations in the levels of labor, consumption, and emissions change with respect to  $\theta$  given the parametrization in Table 3.1. As shown in Section 3.6., variance of labor demand and the consumption of type 2 agent increases with  $\theta$ . The variation of the consumption of type 1 agent declines with  $\theta$ . The variance increases less under the quantity policy; accordingly, relative variance declines. Since both policies are evaluated at the same steady state, the paths of the relative variance in consumption of type 1 and type 2 agents reflect the relative volatility in labor. Social welfare declines with  $\theta$  under both regulations. Note that two policies share the same steady state; the difference between social welfare under the quantity policy relative to the price policy is due to quadratic terms. Despite that the variance is lower under the quantity policy and declines relatively more with  $\theta$ , welfare is higher under the price policy. This stems from the difference in partial derivatives.

The condition in equation 3.93 does not hold in this case. Social welfare is higher under the optimal price policy with respect to the quantity policy, where the quantity is the expected emissions under the optimal tax policy. If the condition were satisfied, it would be enough to show that the optimal quantity policy outperforms the optimal price policy. However, the reverse does not mean that the price policy is better than the quantity policy. Comparison of social welfare under optimal policies necessitates scrutinized optimization routine over quadratic approximations.

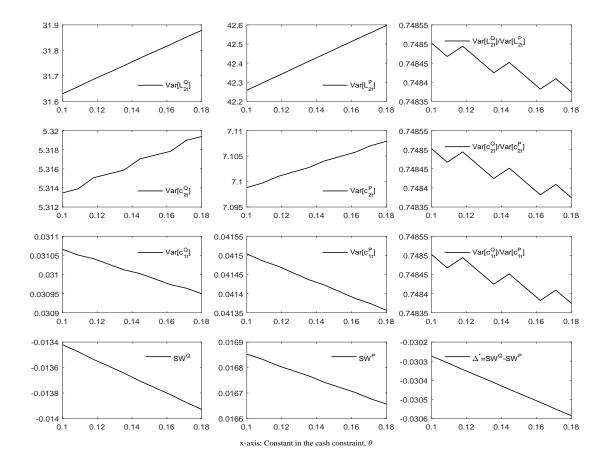


Figure 3.5. Change in Variation in the Economy With Respect to the Degree of the Cash Constraint

## 3.8. Conclusion

In this chapter, we present a heterogeneous agent general equilibrium model that includes a nominal rigidity in the form of partial cash-in-advance constraint in the labor market, pollution externality, and environmental policy. We study macro dynamics in response to a productivity shock under alternative environmental policies. Additionally, we investigate the response of the system to variation in the scale of the cash-in-advance constraint. Two types of environmental policies are considered: price control in the form of fixed real price on pollutant emissions and quantity control in the form of a cap-and-trade system. In the latter case, the government sets a cap on aggregate emissions and issues permits of equal amounts. There is a spot market of emission permits where the government is the sole supplier, and producers demand permits. The producers are obliged to hold emission permits that cover their pollutant emissions.

Our findings are as follows. First, we show that the variance in main macro variables is higher under the price policy than the quantity policy. Response of labor to a positive productivity shock is less pronounced under quantity regulation, as the increased output stimulates a price increase in the permits market, requiring more of output to be reserved for pollution control. This increase in the cost of production weakens the response of labor to productivity shocks.

Furthermore, we demonstrate that as more of labor expenses have to be made in advance, the variation of labor and consumption of the cash-constrained agent increases under both environmental policies, with relatively higher increase under the price policy with respect to the quantity policy. Aligned with the variation in labor, the variation of emissions under price control also rises as the cash constraint becomes more stringent.

The present analysis has prepared the ground for comparing the optimal price and optimal quantity policy. This framework could also be used to study the optimal monetary policy, as we did in the previous chapter. The heterogeneity in the model offers a foundation for integrating the distributional effects of shocks and policies.



# **CHAPTER IV**

# PRICE VS. QUANTITY REGULATION FOR CONTROLLING CAPITAL INFLOWS

# 4.1. Introduction

A surge in capital flows is linked with increased financial and economic instability (Gallagher et al. 2012; Ostry et al. 2011; Ostry et al. 2010). The influx of excessive capital into emerging markets has raised concerns about currency appreciation, escalating asset prices in recipient countries, and excessive borrowing (Gallagher et al. 2012). Research has documented a positive association between capital inflows, credit booms, and foreign exchange lending (Ostry et al. 2011; Mendoza and Terrones 2008). Additionally, it has been observed that excessive holdings of external debt can create balance sheet mismatches, ultimately leading to a balance of payments and debt crises (Furceri et al. 2012; Bordo et al. 2010; Schularick and Taylor 2012; Gourinchas and Obstfeld 2012).

Theoretical analysis indicates that a rise in capital flows, coupled with externalities, can lower welfare. Alongside the empirically documented adverse impacts of capital inflows, the theoretical literature emphasizes externalities as a motivating factor in the implementation of capital controls (see survey articles Dooley 1996; Erten et al. 2021; Rebucci and Ma 2020). These controls aim to maintain the autonomy of monetary policy, preserve the stability of the exchange rate, and control the level and the structure of external debt to limit financial fragility.

Within the context of small open economies, the type of externalities that require capital control measures can be categorized as pecuniary externalities and demand externalities (Erten et al. 2021). Pecuniary externalities arise from the effect of changes in prices on balance sheets when financial markets are imperfect. In the presence of financial imperfections such as collateral constraints on external finance, then change in relative prices (exchange rates or asset prices) bring about an amplification mechanism, imposing welfare costs (e.g. Bianchi 2011; Krugman 1999; Korinek 2011). Aggregate demand externalities emerge due to sticky prices and limitations on macro policies (e.g. fixed exchange rate regime, zero lower bound on monetary policy). Individual agents do not take into account their contribution to aggregate demand generating scope for externalities. This type of externality is subject of the models that focus on monetary policy autonomy (e.g. Farhi and Werning 2012).

The theoretical literature that motivates the prudential use of capital controls focuses on pecuniary externalities and demand externalities (Erten et al. 2021). Learning externalities, where past output determines current productivity (Krugman 1987; Guzman et al. 2018) and the presence of sunk costs (Baldwin and Krugman 1989) also make a case for intervention in capital inflows. Under the presence of sunk costs or learning-by-doing externalities even a temporary overvaluation of the real exchange rate could permanently divert resources out of the tradable sector and undermine welfare. The intensity of these types of externalities may increase due to excessive capital inflows. Although the literature above addresses policy issues associated with these types of externalities, explicit welfare analysis is rare (Benigno and Fornaro 2014).

The effects of capital controls on welfare are studied theoretically (Erten et al. 2021). However, the papers providing cases for the use of capital controls treat price and quantity-based controls as equivalent. They do not compare price and quantity measures; instead they advocate for the use of capital controls without distinguishing between price and quantity controls. As demonstrated by the literature on price versus quantity controls, the equivalence of these measures breaks down when there is uncertainty about the parameters of the economy (Weitzman 1974). Only a few papers address the issue without making an analytic comparison (Erten et al. 2021; Ostry et al. 2011; Magud et al. 2011).

For instance, Ostry et al. (2011) argue that price-based measures are easier to

adjust cyclically but notes that when authorities face information asymmetries and uncertainty about the private sector's response, fixing the price-based measure to achieve the desired quantities can be challenging. According to this paper, quantitybased measures (administrative measures) are susceptible to rent-seeking behavior, and they should be used only if they can be made transparent and rule-based. Another study by Klein (2012) examines the effectiveness of controls on capital inflows by distinguishing controls in terms of their duration. Long-standing controls limit all forms of capital flows at all times, providing isolation but eliminating all potential benefits of capital inflows. Episodic controls, on the other hand, are transitory and targeted towards specific assets. They are not stringent during tranquil times but they are activated in response to capital inflows that cause unwanted appreciation or asset price inflation. Both types have caveats; long-standing controls may eliminate possible benefits, while episodic controls, though benefiting from international capital, may be loose and respond late.

In general, theoretical papers model capital controls as price-based measures. Recognizing that global financial markets are procyclical, prudential regulations on capital inflows that serve as countercyclical measures are considered justified (Gallagher et al. 2012; Korinek 2011). In other words, it is argued that for prudential purposes, tax on capital inflows should be procyclical, increasing during booms and decreasing during downturns (Davis et al. 2021; Aoki et al. 2016; Farhi and Werning 2014; Schmitt-Grohé and Uribe 2012). However, in essence, price controls that are adjusted counter-cyclically along the business cycle are de facto equivalent to fixed quantity controls. A caveat of a tax compared to quantity restriction is that a small tax may not discourage massive inflows (Crotty and Epstein 1996).

The literature on price versus quantity controls explores the optimal mode of regulating a variable. In an important paper, Weitzman (1974) argued that the economic consequences of price and quantity controls are not identical when there is uncertainty concerning the costs and benefits associated with the economic variable that is subject to regulation. In a partial equilibrium setting, the optimal policy depends on the slopes of the marginal benefit and marginal cost schedules when there is uncertainty (Weitzman 1974). This discussion, initially conducted in a partial equilibrium setting, was later extended to a general equilibrium setting with models addressing concerns such as pollution externality. In this paper, we attempt to take this discussion to the realm of controls for capital inflows.

Market-based quantity control mechanisms are applied in the field of pollution control, with cap and trade systems being a notable example (Montgomery 1972). In this setup, government sets a cap on the total allowable amount of pollutants over a predetermined period. A certain amount of credits are allocated to agents, and a penalty is imposed for each unit pollutant emitted beyond the limits. Agents may either choose to reduce their pollutant emissions or purchase emission credits in the secondary market. Notable implementations of this approach include the EU-ETS (European Union Emission Trading Scheme) (Ellerman 2010) and the US Sulfur Dioxide Trading System (Schmalensee et al. 1998).

This chapter aims to make two contributions: First, drawing insights from the literature on price versus quantity controls, we compare welfare implications of price and quantity-type regulation for capital inflows under uncertainty. Second, we point out the concept of a market-based regulatory framework for capital inflows where households require permits to borrow from the rest of the world (Weitzman 1974).

We introduce uncertainty over the global interest rate to the model developed by Benigno and Fornaro (2014, BF), which characterizes a small open economy experiencing endogenous growth. The model incorporates an externality in the form of households not internalizing the growth process in the tradable sector that involves learning by doing. As a result, in the competitive equilibrium, labor allocated to the the tradable sector is less than the socially optimal amount.

Under the price regulation, the regulator's role is to set a tax on external borrowing. With quantity regulation, the regulator sets a cap on aggregate external borrowing and issues borrowing permits, which households can buy in the spot market where the government is the sole supplier, and a single price clears the market. We compare the market mechanism with the tax alternative. In both price and quantity regulation, the regulator sets the policy before observing the interest rate. Agents, however, make decisions after observing the shock. The main questions of interest are: Which mode of regulation yields higher welfare? How does the ranking of policies depend on the initial level of technology (level of development) and the pace of technology growth? We conduct sensitivity analysis with respect to these parameters, as the level of development is a key characteristic for categorizing countries.

We show that under quantity regulation, there is less volatility and, in terms of utility, quantity policy outperforms price policy in the short run. This superiority arises because under price regulation, there is ex-post variation in external debt, and given that agents are risk averse and social welfare is right skewed in external debt. The higher the ex-post variation in external debt, the greater the relative advantage of quantity over price policy in the short term. The ranking of policies is influenced by the initial productivity level, where quantity (price) control performs better in terms of social welfare when the initial productivity level is low (high). The relative advantage of rice over quantity policy declines with an increase in the pace of technology growth.

The chapter is organized as follows. Section 4.2. summarizes the literature on capital flow controls covering their use, effectiveness and justification. Section 4.3. is devoted to presenting the model and the competitive equilibrium under both price and quantity-based regulatory policies. In Section 4.4., we conduct numerical experiments, and finally, Section 4.5 concludes.

# 4.2. Selected Literature on Capital Flows

During the era of the Bretton Woods system, regulations on cross-border capital flows were the norm. The IMF governance of capital flows started as a regime of *cooperative decentralization* (Gallagher 2014). The IMF Articles of Agreement designed at the 1944 United Nations Monetary and Financial Conference in Bretton Woods asserted that no country may restrict current account transactions but granted nations the ability to pursue their own policies to regulate cross border capital flows. Furthermore, the IMF articles encouraged nations to cooperate internationally to enforce such regulations. At the time the articles were framed, similar to the contemporary context, the main motivation behind regulating capital flows was the concern that capital flows may undermine policy autonomy and exchange rate stability. Procyclical capital flows pose the risk of curtailing the government's capacity to expand and counteract the economy when necessary. For instance, an expansionary monetary policy could bring about a capital flight when the economy is in a recession, while a contractionary monetary policy could attract even more capital at a time when the economy is overheating. Procyclical capital flows also exert pressure on the exchange rates.

During the 1960s, industrial nations, including the United States, utilized capital inflow and outflow controls to deal with the balance of payments problems (Gallagher 2014; Crotty and Epstein 1996). This regime of cooperative decentralization began to collapse in the 1970s, with the OECD and IMF advocating for capital account liberalization. OECD codes were amended in 1989 so that short-term flows were liberalized on the grounds that OECD nations had sophisticated enough capital markets (Gallagher 2014). At the same time, capital account liberalization became a prerequisite for OECD accessions. In the 1980s and 1990s, termed as the neoliberal era, capital controls were regarded to be distortionary, leading to the widespread adoption of the notion of capital account liberalization. The underlying premise was that countries receiving capital would benefit from higher growth due to abundant resources and technology transfers, while lenders would benefit from risk diversification and higher returns. In the face of mutually beneficial transactions, the proponent view was unrestricted financial flows. There were pressures to amend IMF articles such that all members commit themselves to the achievement of an open capital account. However, this viewpoint faced opposition from developing countries and economists (Gallagher 2014; Klein 2012). Amidst the system of floating exchange rates following the breakdown of the Bretton Woods, Tobin (1978) addressed the excessive international mobility of private financial capital as an essential problem reducing national policy autonomy. Further, highlighted the severe economic consequences of speculation on exchange rates and proposed the idea of "*throwing sand in the wheels of international money markets*". In particular, he proposed an internationally uniform tax that applies to all purchases of financial instruments denominated in another currency.

The arguments advocating for changes to the IMF Articles lost ground following the Asian financial crises in 1997, for which rapid capital account liberalization and the increase in systemic global financial instability were held responsible for (Akyuz 2002; Crotty and Epstein 1999; UNCTAD 1998). Abundant external financing driving domestic credit expansion played an important role in the emergence of boom-bust cycles. Following the financial crises of 1990, views about the free flow of capital began to change, but the idea of restricting capital flows for prudential purposes was not fully justified until the Global Financial Crises. In December 2012, the IMF adopted a new stance on capital account liberalization and the management of capital flows. This revised perspective acknowledged the inherent risks associated with capital inflows, such as currency appreciation, asset price inflation, and overborrowing. It endorsed the use of regulation on excessive capital inflows with the condition that other macroeconomic policies, foreign reserve management policies, and macro-prudential regulations have been considered (Gallagher and Ocampo 2013). Controls over capital inflows were recognized as preemptive measures to mitigate the risks caused by excessive capital inflows.

The Global Financial Crises of 2008-2009 brought forward the necessity of imposing restrictions on capital flows to reduce the risk created by capital inflows in particular (Erten et al. 2021).<sup>6</sup> The surge in capital inflows raised concerns about currency appreciation, the escalation of asset prices in the recipient countries, and excessive borrowing (Gallagher et al. 2012). Capital flows to emerging markets are more volatile and persistent than advanced economies (IMF 2011). When coupled with low financial strength, these countries are more likely to face financial and macroeconomic instability.

What are the adverse effects of capital inflows? The financial crises of the late 1990s in emerging markets triggered a literature examining the adverse economic consequences of capital flows. Empirical observations have highlighted the association between increased capital market liberalization and both financial and macroeconomic instability (Gallagher et al. 2012; Ostry et al. 2011; Ostry et al. 2010). Studies have documented that domestic firms tend to underprice the risk of external debt and excessive holdings of external debt creates balance sheet mismatches that lead to bankruptcies. Furceri et al. (2012) found that large debt-driven capital inflows increase the likelihood of banking and balance of payment crises. Bordo et al. (2010) observed that a higher share of foreign currency debt is associated with a higher probability of debt crises. Mendoza and Terrones (2008) demonstrated that credit booms and the associated macro fluctuations tend to be more substantial in emerging economies compared to industrial economies. They also noted that credit booms in emerging economies often occur after large capital inflows. Credit flows are associated with the global financial cycle (Rey 2015) and excessive credit growth has been identified as a predictor of crises (Schularick and Taylor 2012; Gourinchas and Obstfeld 2012). Capital inflows, credit booms, and foreign exchange lending are positively associated (Ostry et al. 2011). Capital inflows derive boom-bust cycles. Ostry et al. (2011) point out that 60 percent of the booms that culminate in crises are

<sup>&</sup>lt;sup>6</sup>An extensive survey on capital inflow controls is available in Erten et al. (2021), while Rebucci and Ma (2020) provides a summary of the theoretical and empirical literature following the Global Financial Crises.

associated with a surge in capital inflows.

What justifies the use of capital inflows controls? Alongside the empirically documented adverse impact of capital inflows, the theoretical literature has provided several motivations for the use of capital controls (see survey articles Dooley 1996; Erten et al. 2021; Rebucci and Ma 2020). These include: i) maintaining the autonomy of monetary policy, ii) preserving the stability of the exchange rate as it is believed to influence long-term growth, iii) controlling the level and structure of external debt to limit financial fragility, iv) limiting financial imbalances resulting from speculative attacks by foreign investors (Eichengreen et al. 1995a). Following Guzman et al. (2018), capital controls are tools that serve macroeconomic policy, financial stability, and development.

One of the early arguments for regulating capital flows is the stabilization of output and relative prices. However, pro-cyclical capital flows reduce the space to adopt counter-cyclical policies aimed at securing economic stability (Ocampo 2008). As formulated in the Mundell-Fleming model in the presence of nominal rigidities, a fixed exchange rate eliminates monetary autonomy; in other words, there is no control over the domestic interest rate or money supply. Consequently, with a fixed exchange rate policy, capital controls may be used to regain monetary autonomy (Farhi and Werning 2012; Schmitt-Grohé and Uribe 2012). Taking this argument further, even when the exchange rate is not fixed, changes in relative prices induced by capital inflows may be substantial, again imposing constraint on monetary policy (Rey 2015; Farhi and Werning 2014). Recent research developing around this idea- using models that include nominal rigidities- argues for the benefit of cyclical capital control tax that varies with net capital flow surges and stops (Davis et al. 2021; Aoki et al. 2016; Farhi and Werning 2014; Schmitt-Grohé and Uribe 2012). Given the procyclical nature of global financial markets, prudential regulations on capital inflows that serve as countercyclical measures are acknowledged to be justified (Gallagher et al. 2012;

Korinek 2011).

An important channel through which capital flows bring about economic instability is the real exchange rate. The long-run value of the real exchange rate is expected to be driven by structural factors represented by variables such as trade openness, terms of trade, and productivity differences with respect to trade partners (Alper and Saglam 2000; Combes et al. 2019). The liberalization of capital flows is notably linked with a substantial appreciation of real exchange rates (Dooley 1996). Sizable portfolio inflows can potentially defer adjustments in exchange rates towards the long-run values, resulting in overvaluation and volatility.

There exist both empirical evidence and theoretical explanations, illustrating the influence of capital flows on the real exchange rate, which in turn, affects long-term growth. Guzman et al. (2018) discuss the role of exchange rate policies in promoting economic development and underlines the importance of exchange rate policies in relation to long-term growth. Poor exchange rate management can increase the relative price of non-tradable goods, potentially diverting resources away from the tradable sector and hindering economic diversification and long-term growth. They point out that investment in sectors associated with learning spillovers is suboptimal in unregulated markets. Moreover, they show that a stable and competitive real exchange rate policy may correct this market failure by serving as a subsidy for tradable sectors. Using a panel of 27 EU countries, Comunale (2017) demonstrates that real exchange rate misalignments are associated with lower long-run growth. Combes et al. (2019), employing a sample of low and middle-income countries, reveal that capital inflows undermine growth by impacting the real exchange rate. Furthermore, they show that the instability of foreign direct investments and portfolio investments increase the variation in GDP growth. Li et al. (2018) find a robust association between fund flows and real exchange rate appreciation. The impact of capital flows on the real exchange rate is observed through both the nominal exchange rate and the relative price of non-tradables. Caporale et al. (2017) indicates that nominal exchange rate volatility increases with equity flows. Considering the possibility of hysteresis, Baldwin and Krugman (1989) argues that even a temporary exchange rate appreciation may permanently reduce exports. Under the presence of sunk costs, volatility and persistence in the exchange rate could result in hysteresis in trade and investment Baldwin and Krugman (1989). When there are sunk costs, the hysteresis has important implications for trade, investment, and growth. The firm's position determines its response to an exchange rate shock, with incumbent firms exiting the market if the exchange rate falls below a threshold. Entrant firms, on the other hand, enter the market if the exchange rate rises above a threshold. In the band of inaction, between these thresholds incumbents remain in the market, and potential entrants refrain from entering. Overvaluations in the exchange rate force firms out of the market, and uncertainty widens the bands of inaction. Significant hysteresis losses occur with strong fluctuations and these effects are strongly amplified by exchange rate uncertainty (Belke et al. 2013). In situations where even temporary price changes affect welfare, the free flow of capital becomes undesirable.

Given that large inflows often trigger credit booms and asset market bubbles, another rationale for implementing capital controls is to prevent the accumulation of excessive foreign currency liabilities by both the financial and the real sector. Studies in this line of research examine the role of capital controls as a prudential measure aimed at averting crises or mitigating the severity of financial crises (e.g. Bianchi and Mendoza 2010; Bianchi 2011; Korinek and Sandri 2016). This body of work establishes micro foundations for the potential welfare enhancing effects of capital controls and proposes cyclical capital account regulations.

The notion of capital inflows contributing to heightened financial fragility gained prominence with third generation crises models, which emphasized the lack of international liquidity and balance sheet vulnerabilities as key elements underlying financial distress (Chang and Velasco 1999; Krugman 1999). This way of thinking was followed by theoretical studies providing micro foundations that rationalize the use of capital inflow controls. Within these models, agents overlook the consequences of their choices, leading them to borrow above socially optimal levels (Erten et al. 2021). These externalities induce private agents to accumulate excessive debt, to take excessive risky forms of debt such as borrowing in foreign currency and/or short maturities (Korinek 2011). Consequently, private capital inflows exacerbate the adverse impact of macroeconomic externalities, resulting in an economy characterized by an elevated level of financial fragility.

Concerning small open economies, the externalities requiring capital control measures can be broadly classified as pecuniary externalities and demand externalities (Erten et al. 2021). Pecuniary externalities arise from the impact of price changes on balance sheets when financial markets exhibit imperfections. In the presence of financial imperfections such as collateral constraints on external finance, changes in relative prices (exchange rates or asset prices) trigger an amplification mechanism. For instance, if borrowing capacity depends positively on the real exchange rate level, currency appreciation at the time of capital inflows leads to excessive borrowing. Conversely, during episodes of capital outflow, exchange rate depreciation and declining asset prices reduce collateral and net worth, forcing agents to curtail demand further. This amplification mechanism imposes welfare costs (e.g. Bianchi 2011; Krugman 1999; Korinek 2011).<sup>7</sup>

Aggregate demand externalities arise from sticky prices and limitations on macro policies. In environments that include some form of rigidity, the fact that individual agents do not take into account their impact on aggregate demand creates a ground for externalities. This type of externality is subject of the models focusing on monetary policy autonomy (e.g. Farhi and Werning 2012).

Other types of externalities leading to overborrowing exist. Chang and Velasco (1999) study a model of banking sector in a small open economy with restricted access to international funds and where individual banks fail internalize the impact of their

<sup>&</sup>lt;sup>7</sup>See Korinek (2011) for an overview of models on balance sheet effects and financial amplification.

own borrowing on the country risk ratings. In a model featuring both domestic and international borrowing constraints, Caballero and Krishnamurthy (2001) show that agents undervalue external borrowing due to underdeveloped financial markets and the low return to domestic lending.

These macroeconomic externalities differ from pollution type externalities as their impact on other agents is indirect and conditional on the existence of a friction (e.g. binding borrowing constraints, nominal rigidities). Korinek (2011) categorize externalities into technological externalities and pecuniary externalities. Technological externalities arise when an agents' behavior directly impacts the utility or production function of another agent. Pecuniary externalities indirectly affect other agents by altering relative prices. Changes in relative price per se do not necessitate intervention. If the increase in relative price results in gains for sellers that offset the losses buyers suffer, the new equilibrium is Pareto efficient, and no intervention is required. However, when coupled with frictions, inefficiencies arise. In more general terms, inefficiencies arise when agents in the economy do not behave competitively, or when markets are incomplete.

What measures have been used? So far, we have discussed the adverse effects of capital inflows and the rationale behind using capital controls. In the following, we will briefly summarize the use of capital inflow measures and evaluations related to the effectiveness of these controls. In practice, capital controls are rules, taxes, or fees associated with financial transactions that exhibit discrimination based on residency, distinguishing between domestic residents and those outside the country (Erten et al. 2021). Following the liberalization of capital accounts, several countries in Asia (Indonesia, Malaysia, Thailand), Latin America (Chile, Columbia, Mexico, Brazil), and Europe (Czech Republic) introduced capital inflow controls. These measures were a response to the surge in capital inflows in the beginning of the 1990s and even earlier in the case of Thailand and Malaysia (Magud et al. 2011). The early capital

inflow regulations in Indonesia, Thailand and Malaysia took the form of quantity limits on external borrowing. The Czech Republic also introduced a limit on short-term foreign borrowing by banks. The remaining early measures were price-based, involving taxes on foreign exchange transactions or reserve requirements on external borrowing.<sup>8</sup>

One of the early capital control policies following capital account liberalization was the Chilean *enca je*, enacted in May 1992 and was in effect until May 1998. This policy required a portion of foreign loans to be deposited in a non-interest-bearing account at the central bank. Termed as unremunerated reserve requirement (URR), this policy was also used in Colombia and Thailand in the early 1990s. Brazil introduced a tax on investment in the stock market (Magud et al. 2011). Following the global financial crisis, there was a substantial increase in foreign capital from developed economies to emerging markets. In the face of the adverse effects of excessive inflows, capital controls were introduced or tightened after the Great Recession. For instance, Brazil introduced IOF (the Imposto Sobre Operacoes Financeiras, a tax on investment in existing Brazilian equities), South Korea and Thailand strengthened controls over foreign investors, Peru increased reserve requirements for external borrowing (Klein 2012).

Das and Pugacheva (2020) investigate the motivations behind recalibrating capital controls for the period following the global financial crises using a data set encompassing 11 countries. They show that the likelihood of capital inflow measures increases with capital flow volatility and exchange rate volatility, suggesting that capital flow management (financial imbalances) and real exchange rate rationales are important determinants of capital inflow controls. Furthermore, their results indicate a positive association between the likelihood of capital inflow measures and monetary policy space (defined as short-term interest rate differential vis-a-vis a group of

<sup>&</sup>lt;sup>8</sup>Magud et al. (2011) provide a summary of capital inflow measures by country. See Concha et al. (2011) for a list of capital controls in Colombia and Laan et al. (2017) for an overview of capital controls in Brazil.

economies). Despite the authors' prior expectation of a negative association, the significant positive correlation may reflect the reverse causality, wherein the presence of capital inflow measure enables tighter monetary policy (monetary autonomy).

Were the capital inflow measures effective? Capital inflow controls have proven to be effective in mitigating financial crises. Ostry et al. (2011) document that: i) capital controls are associated with a lower proportion of debt liabilities in total external liabilities; *ii*) Controls on capital flows are associated with a reduction in foreign exchange lending but do not affect lending booms in general; iii) Countries implementing capital controls on debt exhibit greater growth resilience during the Global Financial Crises. Magud et al. (2011) compile results from country specific and multicountry studies assessing the effectiveness of capital inflows. They evaluate whether these controls successfully reduced the net volume of capital inflows, altered the composition of flows towards longer maturities, reduced real exchange rate pressures and made monetary policy more independent. They conclude that, in general, capital controls were successful in altering the composition of flows and providing room for monetary policy. However, the impact of reducing net flows is either short-lived or non-existent except for Chile, Malaysia, and Thailand. The impact on reducing net flows and real exchange pressures is mixed, with Chile standing out as achieving all criteria mentioned above. Malaysia is notable for its success in reducing the net volume of inflows. It is worth emphasizing that Malaysia and Thailand, two out of three countries that were successful in lowering net flows employed quantity-based regulations. Additionally, Ocampo and Palma (2008) point out that short-lived quantitative measures in Malaysia had longer-lasting effects on capital inflows compared to price-based controls in Chile and Colombia.

Empirical studies demonstrate that capital controls have achieved the desired outcomes in cooling economies (Ostry et al. 2011). Using a sample of emerging market economies over the period 1995-2008, Ostry et al. (2012) show that capital controls and foreign exchange-related prudential measures are linked with a lower proportion of foreign exchange-denominated lending domestically and reduced share of portfolio debt in total foreign liabilities. They further emphasize that countries implementing prudential policies and capital controls during the boom tend to exhibit more resilience during periods of outflows. Evaluating the experiences of Malaysia, Chile, and Colombia, Ocampo and Palma (2008) argue that capital account regulations may be instrumental in achieving the policy objective of making economies less susceptible to volatile and unregulated international capital flows. Their reading about the effectiveness of capital controls is that while debt composition is affected, the impact in terms of providing policy autonomy or containing the rise in asset prices tends to be temporary. Focusing on the times of the Great Recession, Laan et al. (2017) show that capital controls were useful in stabilizing portfolio flows. Additionally, Cárdenas and Barrera (1997) conclude that Tobin taxes in Colombia successfully changed the composition of capital inflow in favor of longer maturities, although they did not play a role in changing the overall magnitude of inflows. Coelho and Gallagher (2013) demonstrate that controls effectively reduced total inflows in Thailand and contributed to cooling asset price bubbles in both Thailand and Colombia. In their analysis of the effect of the unremunerated deposit's effect on capital flows in Chile between 1991 and 1998, De Gregorio et al. (2000) find a significant impact on the composition of capital. They note that the effects on the real interest rate and the real exchange rate were transitory. Examining capital inflow controls in Columbia during 1985-1995, Cárdenas and Barrera (1997) determine that these controls had no impact on total flows but successfully altered their composition in favor of long-term flows. Using data from 1993 to 1998, Ocampo and Tovar (1999) show that capital controls reduced the volume of flows and lengthened the maturity of external borrowing. Focusing on controls after 2007 in Colombia, Clements and Kamil (2009) demonstrate that these controls successfully limited external borrowing, but had no significant effect on the volume of non-FDI flows or the currency. In contrast, during the period 1998-2008,

Concha et al. (2011) find capital controls to be ineffective in reducing capital flows and moderating the appreciation of the Colombian peso. Contrary to earlier findings, they find no evidence of a change in the composition of the flows.

What should be the optimal design of capital controls? Currently, capital inflow controls for prudential purposes are considered a justified instrument within the policy set of countries together with macro policies, exchange rate intervention policies, and macro-prudential measures. Nevertheless, there is an ongoing discussion on how to prioritize or combine the use of capital inflow controls with these alternative policies. The IMF view acknowledges the use of capital inflow control measures but conditions it on the nature of flows and the utilization of alternative policy options first. Capital inflow controls are seen as a last resort to be employed only after exhausting other measures. A template provided in Ostry et al. (2010) outlines the optimal response to a surge in capital inflows. Accordingly, controls for macroeconomic reasons, such as appreciation of the exchange rate, should only be used if certain conditions are met. For instance, controls should be considered only if the exchange rate is not undervalued (otherwise allow the currency to appreciate), if reserve accumulation is not necessary (otherwise accumulate reserves), if sterilized interventions in the foreign exchange market are costly (otherwise use sterilized interventions), if there are inflationary concerns (otherwise lower interest rates), and if the fiscal response is not possible (otherwise loosen the budget). Capital controls for prudential concerns, such as avoiding domestic credit boom, are advised only if the prudential regulations prove insufficient. Ostry et al. (2011) further suggests that controls for macroeconomic reasons should only be used in the face of temporary inflows on the grounds that the exchange rate should adjust to permanent shocks.

Capital controls are often compared to sterilized interventions, which involve active intervention in the foreign exchange market and the sterilization of resulting liquidity. Sterilized interventions may be used to substitute or complement capital controls as illustrated by studies such as Davis et al. (2021) and Prasad (2018). However, intervention in the foreign exchange market can be costly when domestic interest rates exceed the return on the reserves. Additionally, there may be an upper bound on the amount of treasury bonds that banks are willing to hold (Xafa 2008). Attempts at sterilized interventions were made in Colombia. However, these were not successful in stopping appreciation of the Colombian peso and incurred quasi-fiscal cost (Coelho and Gallagher 2013).

In Ostry et al. (2011), it is advised to maintain administrative, institutional arrangements in place to facilitate the implementation and adjustment of tax rates on capital inflows. These rates should be increased during booms and reduced when no longer required. When controls are imposed for macroeconomic reasons, a broad application is recommended. However, more targeted controls are advised when the primary concern is financial stability. Ostry et al. (2011) also raise the question whether controls be quantity-based (ceilings, limits) or price-based (tax or URR). Price-based measures are considered easier to adjust cyclically. However, in situations where authorities are uninformed about the nature of the private sector's response to the price policy, it might be challenging to achieve quantity targets by using price-based measures are *preferable in general whereas administrative measures -provided they can be made transparent and rule based (to avoid rent seeking behavior) - may be more appropriate for prudential purposes, particularly when applied to the financial sector(where information asymmetries are particulary relevant)*" (Ostry et al., 2021, pg. 27).

Finally, two important issues related to capital controls need to be addressed. The first issue involves the potential spillover effects of capital controls on other countries (Jeanne 2012; Forbes et al. 2016). This raises concerns about the need for international cooperation in designing capital controls. The question is whether international cooperation can ever really occur (Crotty and Epstein 1996; Eichengreen et al. 1995b). The second issue pertains to the circumvention of capital controls.

Carvalho and Garcia (2008) observe that investors disguise short-term investment as long-term equity and design derivative assets to avoid controls. Ostry et al. (2011) suggest that in countries with sophisticated financial markets, controls and prudential measures could be used together to prevent loopholes.

The optimal design of capital inflow controls constitutes the subject of this chapter. While the aforementioned theoretical papers that provide situations for the use of capital controls treat price and quantity-based controls as equivalent, the literature on price versus quantity controls demonstrates that this duality does not hold when there is uncertainty concerning the parameters of the economy. In this chapter, we compare the welfare implications of price and quantity-based controls. This analysis is conducted within a small open economy's framework where learning-by-doing externalities exist in the tradable sector.

# 4.3. Model

We consider an infinite-horizon small open economy producing tradable and nontradable goods, taking world prices and world interest rates as given. It is populated by a continuum of a unit mass of identical households and by a large number of firms. To study the implications of alternative regulatory frameworks, we introduce a onetime global interest rate shock. We assume a downward shock in the global interest rate at t = 1. Following the first period, the interest rate returns to long-run equilibrium value and remains there for the whole horizon. Uncertainty applies only to the global interest rate of the first period. Households do not internalize that knowledge accumulation depends on the amount of labor allocated to the tradable sector. The role of the government is to handle this friction by managing capital inflows. We assume that the government chooses a policy, shocks are realized, and then households and firms choose allocations. Households and firms act optimally, given the government's policy. In our model costs of accelerating knowledge accumulation is reductions in consumption for which households have concave preferences. The benefit of regulation is higher knowledge stock and consumption in the future.

#### 4.3.a. Firms

Firms operate in tradable and non-tradable sectors. In both sectors, there are large number of firms that produce using labor as a single factor of production. In the tradable sector, firms combine labor,  $L_t^T$ , with the stock of knowledge  $A_t$  (total factor productivity, TFP) according to the production function:

$$Y_t^T = A_t L_t^T, (4.2)$$

where  $Y_t^T$  is the amount of tradable goods produced at time *t*. Knowledge is non-rival and non-excludable, and so it is free. Firms face labor expenses  $w_t L_t^T$ . Firms produce up to the point where marginal productivity equals marginal cost, implying that real wage equals the TFP level, i.e.,  $w_t = A_t$ . Knowledge accumulation is an endogenous process. The stock of knowledge evolves according to

$$A_{t+1} = A_t \left[ 1 + cL_t^T \left( 1 - \frac{A_t}{A_t^*} \right) \right], \qquad (4.3)$$

where c > 0 is a parameter characterizing the impact of labor on knowledge growth.  $A_t^*$  denotes the stock of knowledge at the frontier, which grows at the constant rate  $g^*$ . Human capital facilitates the catchup process and ensures a higher stock of capital at the steady state. In the steady state, where technology at home and the frontier grow at the same rate  $g^*$ , proximity to the frontier is:

$$\frac{A_t}{A_t^*} = 1 - \frac{g^*}{cL_t^T}$$
(4.4)

This equation indicates that in the steady state, the stock of knowledge is closer to the frontier if more workers are employed in the tradable goods sector.

In the non-tradable goods sector, firms produce using only labor according to

the production function  $Y_t^N = L_t^N$ . Profits in the tradable sector are maximized at the point where the relative price of non-tradables is equal to the real wage, i.e.,  $p_t^N = w_t$ . Combining this with the optimality condition in the tradable goods sector, we obtain

$$p_t^N = A_t. \tag{4.5}$$

# 4.3.b. Government

The government sets the regulatory policy for the first period only. At the beginning of date 1, the government determines the regulatory policy either in the form of a tax ( $\tau_t$ ) or a cap on foreign debt ( $X_t^s$ ), then shocks are realized, and agents make borrowing and first-period consumption decisions. Due to the timing described, only the government faces uncertainty, whereas households make decisions after observing the shock. At date 2, the global interest rate returns to long-run equilibrium value and remains there for the whole horizon. There is no uncertainty in the model except for the first period.

# 4.3.c. Households' Problem Under Price Control

Representative household derives utility from consumption and supplies inelastically L units of labor. Household faces the following problem:

$$\max \sum_{t=1}^{\infty} \beta^{t} u(c_{t}) \text{ subject to, for all t}$$
(4.6)

$$c_t = \left(c_t^T\right)^{\omega} \left(c_t^N\right)^{1-\omega} \tag{4.7}$$

where  $\omega \epsilon$  (0,1) denotes the share of non-tradables in consumption. The budget constraint of the household is

$$c_t^T + p_t^N c_t^N + \frac{B_{t+1}}{R_t (1 + \tau_t)} = w_t L + B_t + \Pi_t + TR_t.$$
(4.8)

The price of tradable goods is the numeraire. The budget constraint is expressed in units of tradable goods.  $p_t^N$  is the relative price of non-tradable goods at time t, and  $w_t$  is the wage rate.  $\Pi_t$  stands for profits of the firms,  $B_t$  represents the net foreign assets of the household.  $\frac{B_{t+1}}{R_t}$  is the present value of net foreign assets in the next period.  $R_t$  is the gross world interest rate,  $\tau_t$  is the tax rate on capital inflows, and  $TR_t$  stands for lump-sum government transfers to the households. The right-hand side of (4.8) represents the income of the households.

The gross world interest rate is random in the first period but is fixed for the rest of the horizon.

$$R_t = \begin{cases} \sim N(\mu_R, \sigma_R) & \text{for } t = 1 \\ R & \text{for } t > 1 \end{cases}$$
(4.9)

*R* is the long-run value of the global rate. Above, we suppose that the first-period interest rate is distributed normally with mean  $\mu_R$  and standard deviation  $\sigma_R$  such that  $\mu_R < R$ .

At each period, the representative household chooses  $c_t^T$ ,  $c_t^N$ , and  $B_{t+1}$  to maximize utility subject to the budget constraint. The first-order conditions are:

$$\frac{\omega}{c_t^T} = \lambda_t \tag{4.10}$$

$$\frac{1-\omega}{c_t^N} = \lambda_t p_t^N \tag{4.11}$$

$$\lambda_t = \beta R_t (1 + \tau_t) \lambda_{t+1} \tag{4.12}$$

Combining (4.10) and (4.11) we get the equilibrium condition that sets the relative price of non-tradable goods equal to the marginal rate of substitution between tradable and non-tradable goods:

$$\frac{1-\omega}{\omega}\frac{c_t^T}{c_t^N} = p_t^N \tag{4.13}$$

The third first-order condition (4.12) is the Euler equation that determines the intertemporal allocation of consumption between consecutive periods.

### 4.3.d. Competitive Equilibrium Under Price Regulation

Market clearing in the non-tradable goods sector requires

$$c_t^N = Y_t^N. ag{4.14}$$

Combining market clearing conditions in the non-tradable sector with households' budget constraint, firms' optimality conditions, and equations for firms' profits, we obtain market clearing conditions for the tradable sector as

$$c_t^T = Y_t^N - \frac{B_{t+1}}{R_t(1+\tau_t)} + B_t + TR_t.$$
(4.15)

In the equilibrium, the labor market clears

$$L = L_t^T + L_t^N. aga{4.16}$$

The set of prices and quantities  $\{p_t^N, w_t, p_t^x, c_t^T, c_t^N, L_t^T, L_t^N, Y_t^T, Y_t^N, B_{t+1}, X_t^d, A_{t+1}\}_{t=0}^{\infty}$  constitute competitive equilibrium if,

1. Quantities  $\{c_t^T, c_t^N, L_t^T, L_t^N, B_{t+1}\}_{t=0}^{\infty}$  solve the constrained optimization problem of the households under the sequence of prices  $\{p_t^N, w_t, p_t^x\}_{t=0}^{\infty}$  and exogenous processes  $\{R_t, A_t^*, \tau_t\}_{t=0}^{\infty}$  and initial conditions  $B_0$  and  $A_0$ . Assuming  $u(c_t) = \ln(c_t)$ , the following conditions are derived from households' optimization problem:

$$\frac{1-\omega}{\omega}\frac{c_t^T}{c_t^N} = p_t^N \tag{4.17}$$

$$c_{t+1}^{T} = \beta R_t (1 + \tau_t) c_t^{T}$$
(4.18)

$$c_{t+1}^{N} = \beta R_t (1 + \tau_t) c_t^{N} \frac{A_t}{A_{t+1}}$$
(4.19)

2. Quantities  $\{L_t^T, L_t^N\}_{t=0}^{\infty}$  solve the constrained optimization problem of the firms under the sequence of prices  $\{p_t^N, w_t\}_{t=0}^{\infty}$  and the stock of knowledge  $A_{t}$ .

$$w_t = A_t = p_t^N. aga{4.20}$$

3. Labor markets clear for all t

$$L = L_t^T + L_t^N. aga{4.21}$$

4. Non-tradable goods market clears for all t

$$c_t^N = Y_t^N = L_t^N. (4.22)$$

5. Government budget is balanced for all t. All tax revenues are transferred back to the agents.

$$TR_t = -\frac{\tau_t B_{t+1}}{R_t (1 + \tau_t)}.$$
(4.23)

6. Tradable goods market clears for all t

$$c_t^T = Y_t^T - \frac{B_{t+1}}{R_t(1+\tau_t)} + B_t + TR_t.$$
(4.24)

7. Stock of knowledge grows according to

$$A_{t+1} = A_t \left[ 1 + cL_t^T \left( 1 - \frac{A_t}{A_t^*} \right) \right].$$

$$(4.25)$$

8. Transversality condition is satisfied.

$$\lim_{T \to \infty} \frac{B_T}{R_{T-1}} = 0.$$
(4.26)

Combining equations (4.17) and (4.20)-(4.24), labor allocated to the tradable sector and the consumption of tradables are given by:

$$L_{t}^{T} = \omega L + \frac{1 - \omega}{A_{t}} (\frac{B_{t+1}}{R_{t}} - B_{t})$$
(4.27)

$$c_t^T = \omega (AL - \frac{B_{t+1}}{R_t} + B_t) \tag{4.28}$$

If there are no financial markets, labor allocation reflects the consumption preferences of the household. If households can borrow on the other hand,  $B_{t+1} < 0$ , they bring forward consumption and allocate less labor to the tradable sector.

### 4.3.e. Households' Problem Under Quantity Regulation

Households get utility from consumption. There is no utility from leisure, hence L units of labor is supplied inelastically. Households face the following problem:

$$\max \sum_{t=1}^{\infty} \beta^t u(c_t) \text{ subject to, for all t}$$

$$c_t = \left(c_t^T\right)^{\omega} \left(c_t^N\right)^{1-\omega} \tag{4.29}$$

where  $\omega \varepsilon (0,1)$  denotes the share of non-tradables in consumption. We assume that taste shocks are good specific disturbances to the utility at each date t. The budget constraint of the household is

$$c_t^T + p_t^N c_t^N + \frac{B_{t+1}}{R_t} + p_t^X X_t^d = w_t L + B_t + \Pi_t + TR_t, \qquad (4.30)$$

$$-B_{t+1} \le X_t^d. \tag{4.31}$$

with

$$R_t = \begin{cases} \sim N(\mu_R, \sigma_R) & \text{for } t = 1 \\ R & \text{for } t > 1 . \end{cases}$$
(4.32)

The price of tradable goods is the numeraire. The budget constraint is expressed in units of tradable goods.  $p_t^N$  is the relative price of non-tradable goods at time t.  $w_t$  is the wage rate.  $\Pi_t$  stands for profits of the firms.  $B_t$  represents assets of the household.  $\frac{B_{t+1}}{R_t}$  is the present value of assets in the next period.  $R_t$  is the gross world interest rate.  $p_t^X$  is the relative price of borrowing permits at time t, and  $X_t^d$  is the number of permits demanded by the household.  $TR_t$  stands for lump-sum government transfers to the households. The right-hand side of (4.32) represents the income of the households.

At each period households choose  $c_t^T$ ,  $c_t^N B_{t+1}$  and  $X_t$  to maximize utility subject to the budget and borrowing constraints. The first-order conditions are:

$$\frac{\omega}{c_t^T} = \lambda_t \tag{4.33}$$

$$\frac{1-\omega}{c_t^N} = \lambda_t p_t^N \tag{4.34}$$

$$\lambda_t = \beta R_t \lambda_{t+1} + \beta R_t \gamma_t \tag{4.35}$$

$$\lambda_t p_t^x = \gamma_t \tag{4.36}$$

Combining (4.33) and (4.34) we obtain the equilibrium a condition that sets the relative price of non-tradable goods equal to the marginal rate of substitution between tradable and non-tradable goods:

$$\frac{1-\omega}{\omega}\frac{c_t^T}{c_t^N} = p_t^N \tag{4.37}$$

Combining (4.35) and (4.36) we obtain the the Euler the equation that determines the intertemporal allocation of consumption between consecutive periods

$$\lambda_t (1 - R_t p_t^x) = \beta R_t \lambda_{t+1}. \tag{4.38}$$

The firms' problem is the same as in the model with price control.

# 4.3.f. Competitive Equilibrium Under Quantity Regulation

The set of prices and quantities  $\{p_t^N, w_t, p_t^x, c_t^T, c_t^N, L_t^T, L_t^N, Y_t^T, Y_t^N, B_{t+1}, X_t^d, A_{t+1}\}_{t=0}^{\infty}$  constitute competitive equilibrium if,

Quantities {c<sub>t</sub><sup>T</sup>, c<sub>t</sub><sup>N</sup>, L<sub>t</sub><sup>T</sup>, L<sub>t</sub><sup>N</sup>, B<sub>t+1</sub>, X<sub>t</sub><sup>d</sup>}<sub>t=0</sub><sup>∞</sup> is a solution to the constrained optimization problem of the household under the sequence of prices {p<sub>t</sub><sup>N</sup>, w<sub>t</sub>, p<sub>t</sub><sup>x</sup>}<sub>t=0</sub><sup>∞</sup> and exogenous processes {R<sub>t</sub>, A<sub>t</sub><sup>\*</sup>, X<sub>t</sub>} and initial conditions B<sub>0</sub> and A<sub>0</sub>. Assuming u(c<sub>t</sub>) = ln(c<sub>t</sub>) and that the borrowing constraint is binding at all times, the following conditions are derived from households' optimization problem:

$$\frac{1-\omega}{\omega}\frac{c_t^T}{c_t^N} = p_t^N \tag{4.39}$$

$$c_{t+1}^{T} = \frac{\beta R_t}{(1 - R_t p_t^x)} c_t^{T}$$
(4.40)

$$X_t = -B_{t+1} \tag{4.41}$$

2. Quantities  $\{L_t^T, L_t^N\}_{t=0}^{\infty}$  are a solution to the constrained optimization problem of the firms under the sequence of prices  $\{p_t^N, w_t\}_{t=0}^{\infty}$  and the stock of knowledge  $A_t$ .

$$w_t = A_t = p_t^N. ag{4.42}$$

3. Market on borrowing permits clears for all t. The equilibrium price of permits the outcome of optimal household behavior.

$$X_t^d = X_t^s. \tag{4.43}$$

4. Labor markets clear for all *t*.

$$L = L_t^T + L_t^N. aga{4.44}$$

5. Non-tradable goods market clears for all t.

$$c_t^N = Y_t^N. (4.45)$$

6. Government budget is balanced for all *t*. All tax revenues are transferred back to the agents.

$$p_t^X X_t^s = T R_t. ag{4.46}$$

7. Tradable goods market clears for all t.

$$c_t^T = Y_t^T - \frac{B_{t+1}}{R_t} + B_t - p_t^X X_t + TR_t.$$
(4.47)

8. Stock of knowledge grows according to

$$A_{t+1} = A_t \left[ 1 + cL_t^T \left( 1 - \frac{A_t}{A_t^*} \right) \right].$$
 (4.48)

9. Transversality condition is satisfied.

$$\lim_{T \to \infty} \frac{B_T}{R_{T-1}} = 0.$$
(4.49)

Under quantity regulation, the government sets the cap on the second period's debt. Therefore  $B_2$  is pinned down by the government, so is  $L_1^T$  and the consumption basket of the first period. This can be derived by looking at the equations (4.27) and (4.28). After the realization of the interest rate shock at t = 1,  $p_1^x$  and  $\{c_t^T\}_{t=2}^{\infty}$  adjust to satisfy the equilibrium conditions.

We present the summary of notations used in our model and the following sections in Table 4.1 below.

#### 4.4. Numerical Experiments

In this section, we study the dynamics of the model using numerical experiments. Similar to the approach in BF, our analysis begins with the economy positioned below the steady state productivity level in the tradable sector. As technology advances, the economy gradually converges to a steady state where eventually technology growth aligns with the frontier. To explore the implications of alternative regulatory frameworks, we introduce a one-time global interest rate shock. Specifically, we assume a downward shock in the global interest rate at t = 1. Subsequently, the global rate

Notation	Description
t	The index of time
$Y_t^T$	The amount of tradable goods produced
$L_t^T$	The amount of labor in the tradables sector
$L^{T} \\ Y_{t}^{T} \\ L_{t}^{T} \\ L_{1}^{T,P} \\ L_{1}^{T,P}$	The amount of labor in the tradables sector
	under optimal price regulation at $t = 1$
$L_1^{T,Q}$	The amount of labor in the tradables sector
1	under optimal quantity regulation at $t = 1$
$Y_t^N$	The amount of non-tradable goods produced
$ \begin{array}{l} Y_t^N \\ L_t^N \\ p_t^N \\ B_t \end{array} $	The amount of labor in the non-tradables sector
$p_t^N$	The relative price of non-tradable goods
$B_t$	The net foreign assets of the households
$\Pi_t$	The profits of the firms
	The tax rate on foreign debt
$egin{array}{ccc}  au_t & & \  au_t^* & & \  au_t^T & & \  au_t^T & & \  au_t^N & & $	The optimal tax rate under price regulation
$\dot{c_t^T}$	Consumption of tradable goods
$c_t^N$	Consumption of non-tradable goods
$\dot{T}R_t$	Lump-sum government transfers to the households
W <sub>t</sub>	The real wage
β	The discount factor
Ĺ	The endowment of labor
$B_t$	Net foreign assets at t
$A_t$	TFP of the small open economy at time t
$\begin{array}{c} A_t \\ A_t^* \\ g^* \end{array}$	TFP of the technological leader at time t
$g^*$	The growth rate of TFP at the frontier
$d_t$	Proximity to the frontier
с	The constant in knowledge accumulation process
ω	The share of tradable goods in consumption
$p_t^x$	The relative price of borrowing permits
$X_t^d$	The amount of borrowing permits demanded by the household
$p_t^x$ $X_t^d$ $X_t^s$ $X^*$	The amount of borrowing permits supplied by the government
$X^*$	The optimal cap on borrowing permits under quantity regulation
k	The index of the round number in the grid search
$Lb_k$	The lower bound of the interval in the grid search
$Ub_k$	The upper bound of the interval in the grid search
η	The welfare gain
BF	the abbreviation for benchmark framework
NF	the abbreviation for new framework

 Table 4.1. Summary of Notations in Chapter IV

immediately reverts to its long-term equilibrium value and remains there for the rest of the time.

Building on BF's findings, which demonstrate that regulation on capital flows improves welfare due to growth externality, we compare the effects of alternative regulatory policies. In this section, we show that, under uncertainty over the size of the global liquidity shock, quantity and price policies have different implications on transition dynamics and welfare. Additionally, we observe that welfare gains from price policy over quantity policy decline with the increase in the impact of labor allocation on knowledge accumulation (c) and with the drop in the initial stock of TFP ( $A_1$ ).

In this section, we carry out the following numerical exercises: *i*) Initially, we replicate the transition dynamics of the model under the original setup in BF, *ii*) subsequently, we introduce one-period shock and document the resulting changes in the transitory dynamics without implementing any regulatory policy, *iii*) as a third exercise, we introduce optimal price and quantity policy to study the transition under two different episodes for the realization of the interest rate: one with  $R_1 > \mu_R$  and the other with  $R_1 < \mu_R$ , *iv*) next we compare expected welfare gains of optimal price and quantity policies for alternative parametrizations of the model. In particular, we explore how welfare under optimal tax and quantity policies responds to different constants in the knowledge accumulation process and varying initial TFP levels. Our findings reveal that the preference for regulatory framework changes based on the initial TFP, with quantity policy being favored over price policy for low TFP levels and vice versa for high levels of initial TFP.

In this section, we also compare the volatility in main variables of interest under price and quantity policies. Before delving into the results of our numerical experiments, we explain the technical aspects involved in solving the model and identifying optimal regulatory policies.

#### 4.4.a. Solving the Model and Identifying Optimal Policies

**Solving the Model** We employ the algorithm outlined in BF to solve the model (Benigno and Fornaro 2014).<sup>9</sup> Initially, we estimate a starting value for the consumption of the traded good. We solve the model using this estimate and check whether the transversality condition is met. We iteratively update the first-period consumption of the traded good until the condition is satisfied.

In the scenarios without a regulatory framework or when price regulation is in effect, the algorithm begins by setting a value for consumption of the traded good from the first period. However, under the framework where quantity regulation is in effect, the first-period consumption of the traded good is pinned down by the cap on second-period debt. In this case, the algorithm starts from the second period, given the outcome of the first period.

**Identifying Optimal Regulatory Policies** We use grid search to identify optimal regulatory policies. The optimal tax rate and optimal cap on foreign debt are determined such that social welfare, represented by the discounted utility as defined in (4.6), is maximized under the competitive equilibrium. Initially, we use fine grid search over  $\tau$  to identify optimal tax policy. In the first round (k = 1) we start with a wide interval for  $\tau$  with *n* evenly spaced points between the lower bound  $Lb_{k=1}$  and the upper bound  $Ub_{k=1}$  and find the optimal rate  $\tau_{k=1}^*$ . If  $\tau_{k=1}^*$  hits the boundaries, the grid is updated such that  $\tau_{k=1}^*$  is the midpoint and the interval size is the same. In this case, the interval is updated according to the following rule:

$$Lb_{k+1} = \tau_k^* - (Ub_k - Lb_k)/2 \tag{4.50}$$

$$Ub_{k+1} = \tau_k^* + (Ub_k - Lb_k)/2 \tag{4.51}$$

If  $\tau_{k=1}^*$  is an interior solution we define a narrower band around  $\tau_{k=1}^*$  with *n* evenly

<sup>&</sup>lt;sup>9</sup>I am thankful to Luca Fornaro for sharing the codes of the optimization routine used in their paper.

spaced points between  $Lb_{k=2}$  and  $Ub_{k=2}$ , making sure that all potential rates fall within the band. In this case, the interval is updated at each round according to the following rule:

$$Lb_{k+1} = \tau_k^* - (Ub_k - Lb_k)/(n-1)$$
(4.52)

$$Ub_{k+1} = \tau_k^* + (Ub_k - Lb_k)/(n-1)$$
(4.53)

This process continues up to the point where the difference between  $\tau_k^*$  and  $\tau_{k-1}^*$  is below the tolerance limit and  $\tau_k^*$  is an interior solution. We use an iterative algorithm that is complete and that gives optimal policy that is accurate at 4-digit level. The algorithm is complete in the sense that all possible values for the policy parameter are covered.

Once we have an estimate for  $\tau^*$ , we compute the expected holdings of foreign debt under the optimal tax rate. Using this value as the midpoint, we apply the same algorithm to determine the optimal cap on the foreign debt level. The tolerance limit for the cap on foreign debt is aligned with the tolerance limit on the tax rate. Specifically, it is equal to the change in foreign debt induced by a change in  $\tau$  at the size of the tolerated amount. This choice ensures the consistency of the grids over the tax on external debt ( $\tau$ ) and the supply of borrowing permits ( $X^s$ ).

# Figure 4.1. Identifying Optimal Regulatory Policies

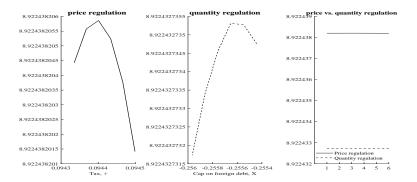


Figure 4.1 illustrates how social welfare responds to  $\tau$  and  $X^s$  in the final round of the grid search. The first panel depicts the expected social welfare response to the tax rate under price regulation, while the second panel shows the response to the cap on foreign debt under quantity regulation. The third panel compares expected social welfare under price and quantity policy at the final grid. Notably, under both regulatory frameworks, optimal policies are the interior solution of the maximization routine. This figure illustrates how we identify optimal policies, a methodology consistently applied throughout the paper to identify optimal policies under alternative model parametrizations. In the subsequent sections, we present the results of the numerical exercises.

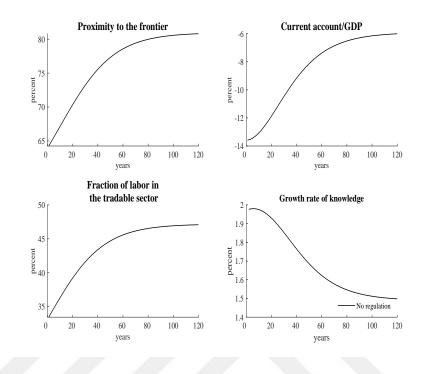
### 4.4.b. Results of the Numerical Experiments

In most of our numerical experiments, we adopt the original parametrization of the BF model, as outlined in Table 4.2. Except the values characterizing the distribution of the interest rate at t = 1, all other parameters in the table remain identical to those in the BF model. Simulations are conducted based on the original parametrization of the model unless explicitly stated otherwise.

Parameter	Symbol	Value
Growth rate of the technological frontier	$g^*$	0.015
World interest rate at $t = 1 \sim N(\mu_R, \sigma_R)$	$(\mu_R, \sigma_R)$	(1.02, 0.005)
World interest rate $t > 1$	R	1.04
Discount factor	β	0.976
Endowment of labor	L	1
Initial NFA	$B_0$	0
Initial TFP of the technological leader	$A_0^*$	6.4405
Initial TFP	$A_0$	4.1384
Constant in knowledge accumulation process	С	0.167
Share of tradable goods in consumption	ω	0.414

Table 4.2. Parameter Set of Chapter IV





**Transition toward the steady state without regulatory policy** Figure 4.2 depicts the transition toward the steady state in the competitive equilibrium without policy intervention. At the initial stage, the economy starts below its steady state proximity to the frontier, resulting in a higher growth rate of knowledge compared to the world technological leader. Over time, as the stock of knowledge approaches the frontier, the growth rate of knowledge converges to that of the world leader. During this convergence process, labor allocated to the tradable sector increases. As the stock of knowledge increases, hiring labor in the tradable sector becomes more profitable. As labor shifts towards the tradable sector, production and consumption in the non-tradable sector declines, increasing the relative price of non-tradable sector. These opposing forces balance each other in the steady state, resulting in a constant level

of labor in the tradable sector. During the transition, households accumulate foreign debt, generating higher current account deficits compared to the steady state. This occurs because, along the convergence process, the output of tradables grows faster than in the steady state. Consumers, driven by the desire to smooth consumption, allocate consumption towards earlier stages of the period, contributing to the observed accumulation of foreign debt.

Figure 4.3. Transition Toward the Steady State: Benchmark vs. One-Period Global Interest Shock

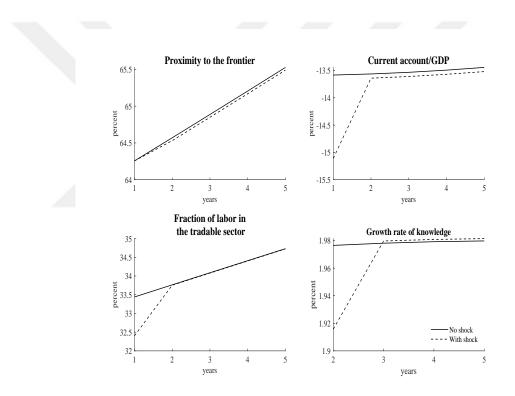
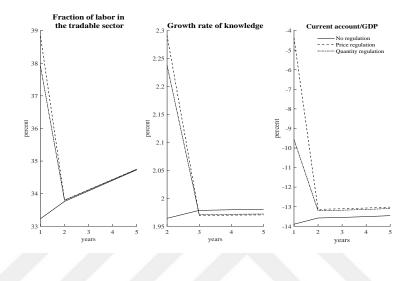


Figure 4.3 compares transition under the base scenario (with no shock) with an episode involving a one-period drop in the interest rate, represented by the dashed lines ( $R_1 = \mu_R$ ,  $R_t = R$  for t > 1). In response to a decrease in the interest rate, households seek to boost consumption in the earlier periods. Hence, they borrow more from abroad and allocate less labor to the tradable sector, aiming to increase

the production and consumption of non-tradable goods. In this case, the speed of convergence is lower initially, and proximity to the frontier remains at a lower level for an extended period along the transition path.



**Figure 4.4.** Transition Toward the Steady State Under Optimal Policy,  $R_1 > \mu_R$ .

**Transition toward the steady state under optimal regulatory policy** In this part, we simulate the stochastic version of the model where  $R_1 \sim N(\mu_R, \sigma_R)$ . The government sets optimal regulatory price and quantity policy based on expectations over the interest rate. The optimal policy, set according to expected social welfare, remains fixed. Figure (4.4) shows the transition to the steady state under a non-regulatory framework, optimal price policy, and optimal quantity policy, providing a comparative context when  $R_1 > \mu_R$  (Figure 4.6 presents the same for  $R_1 < \mu_R$ ). Dynamics differ depending on whether  $R_1 > \mu_R$  or  $R_1 < \mu_R$ , and therefore we describe these as separate cases. The relative positioning of the transition paths for the variables changes under alternative policies, depending on the realization of the interest rate shock.

The regulation applies only in the initial period. Under price regulation, the tax

rate is fixed, and debt becomes an increasing function of the interest rate (Figure 4.5). In contrast, under quantity regulation, there is a cap on foreign debt, and it is costly to hold debt as households must purchase permits equivalent to the amount of debt. The price of permits is a decreasing function of the realization of the interest rate (Figure 4.5). The regulation amplifies the cost of consumption today, prompting households to consume less. Consequently, they allocate more labor to the tradable sector, resulting in reduced borrowing and a higher growth rate of knowledge during the initial period.

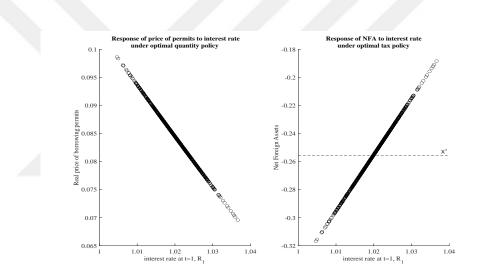
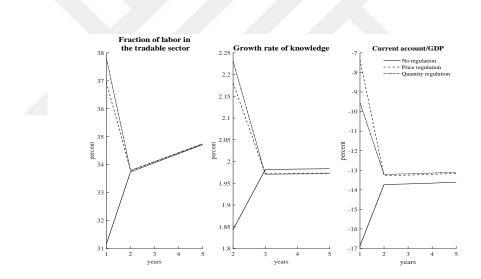


Figure 4.5. Adjustment Margins Under Regulatory Price and Quantity Policies

Figure 4.4 shows the ex-post dynamics of the model when  $R_1 > \mu_R$ . The panels depict the response of the fraction of labor in the tradable sector, the growth rate of knowledge, and the current account/GDP to a positive interest rate shock. The solid line represents dynamics under a non-regulatory environment, while the dashed and dash-dotted lines show transition under optimal price and optimal quantity policies. The fraction of labor in the tradable sector increases with the interest rate and foreign assets as defined in equation (4.27). In the first period, both regulatory frameworks result in higher fractions of labor in the tradable sector and a faster growth rate of knowledge compared to the non-regulatory environment. However, these effects are even more pronounced under price regulation. This occurs because, under price regulation, the ex-ante optimal tax rate turns out to be higher than optimal, leading to a downward adjustment in external debt in response to a positive interest rate shock. This further increases the fraction of labor in the tradable sector. No such adjustment occurs under quantity regulation, as external debt is fixed, and labor responds only to the interest rate. Therefore, in the first period, labor in the tradable sector is higher under price than quantity regulation. Along with higher labor in the tradable sector, debt is lower, and technology growth is higher in the first period. This higher technology growth ensures that labor in the tradable sector is lower in the subsequent periods.

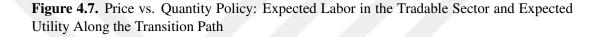


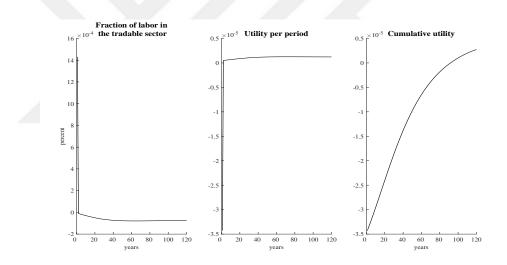
**Figure 4.6.** Transition Toward the Steady State Under Optimal Policy,  $R_1 < \mu_R$ .

Figure 4.6 shows the ex-post dynamics of the model when  $R_1 < \mu_R$ . Under this episode, the fraction of labor in the tradable sector is lower under price regulation. This is attributed to the ex-ante optimal tax rate being lower than the optimal rate, leading to a shift in labor towards the non-tradable sector, accompanied by increased borrowing by households. Under quantity regulation, labor in the tradable sector responds only to negative interest rate shock. In contrast, under price regulation,

external debt increases further, reducing the share of labor in the tradable sector. In the first period, households, constrained by the fixed level of foreign debt under quantity regulation, have no flexibility. Consequently, in the first period, labor in the tradable sector is lower under price regulation compared to quantity regulation.

Welfare under alternative regulatory policies. Up to this point, we have examined the dynamics of the non-stochastic model or the ex-post dynamics of the stochastic model. From this point forward, we will focus on the ex-ante expected model dynamics under alternative regulatory policies. In Figure 4.7, we compare the transition





under optimal price policy with that under optimal quantity policy. The figures represent expected gaps. The first panel illustrates the gap in expected labor share in the tradable sector, i.e.  $E(L_t^{T,P}) - E(L_t^{T,Q})$ , where *P* and *Q* represents price and quantity policies, respectively. The second panel displays the gap in expected period utility, i.e.  $E(u_t^P) - E(u_t^Q)$ , and the third panel shows the gap in expected discounted cumulative utility, i.e.  $E(cu_t^P) - E(cu_t^Q)$ . Cumulative utility as of time *t* is given by,

$$cu_t = \sum_{s=1}^t \beta^{s-1} u(c_s).$$
 (4.54)

In the initial period, the expected fraction of labor in the tradable sector is higher under price regulation. This is due to the optimal cap on foreign debt being higher than the expected debt under optimal tax policy. In the following, we explain the reason behind this observation. Using equation (4.27) and assuming that  $B_1 = 0$ , the expected share of labor under price (*P*) and quantity (*Q*) regulations is given below:

$$E(L_1^{T,P}) = \omega L + \frac{(1-\omega)}{A_1} E\left(\frac{B_2^p(\tau^*)}{R_1}\right)$$
(4.55)

$$= \omega L + \frac{(1-\omega)}{A_1} \left[ E\left(B_2^P(\tau^*)\right) * E\left(\frac{1}{R_1}\right) + cov\left(B_2^P, \frac{1}{R_1}\right) \right]$$
(4.56)

$$E(L_1^{T,Q}) = \omega L + \frac{(1-\omega)}{A_1} X_1^* E\left(\frac{1}{R_1}\right)$$
(4.57)

The gap is given by:

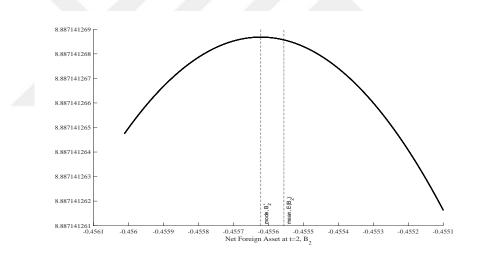
$$E(L_1^{T,P}) - E(L_1^{T,Q}) = E\left(\frac{1}{R_1}\right) \left[E\left(B_2^P(\tau^*)\right) - X^*\right] + cov\left(B_2^P, \frac{1}{R_1}\right)$$
(4.58)

The expected labor in the tradable sector is higher under price regulation relative to quantity regulation when the following conditions are met.

$$X^* - E(B_2^P(\tau^*)) < \frac{cov(B_2^P, \frac{1}{R_1})}{\frac{1}{R_1}}$$
(4.59)

 $B_2^P$  increases with the interest rate. Therefore, the right-hand side of the above inequality is less than zero. A necessary condition for  $E(L_1^{T,P}) > E(L_1^{T,Q})$  is that  $X_1^* < E(B_2^P(\tau^*))$ . Next, we show that, under our model setup, the expected external debt under optimal price regulation is higher than the optimal cap on external debt set under quantity regulation. This stems from the uncertainty over  $B_2$  under price regulation and the shape of the social welfare function. Social welfare is concave in  $(B_2)$  and exhibits right skewness around the optimum level of net foreign assets (corresponding to left skewness in foreign debt). Figure 4.8 depicts social welfare as a function of net foreign assets  $(B_2)$  in a setup when there is no regulatory policy and no uncertainty over the global interest rate. External debt increases from left to right on the horizontal axis, and social welfare follows an inverse U-shaped pattern in external debt. It increases with external debt for at low levels of debt and declines slightly faster beyond the maximum. An optimal tax level corresponds to a vector of realizations of  $B_2$ , which constitutes the support of the social welfare function. Evaluating the function  $SW(B_2)$  for this support reveals that when the function is right skewed, the maximizer lies below the mean of the support.

**Figure 4.8.** Response of Social Welfare to NFA When There is No Uncertainty Over the Global Interest Rate



Note: Social welfare is the discounted utility as in equation (4.6).

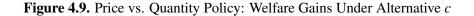
Going back to figure 4.7, the expected fraction of labor in the tradable sector is lower under price regulation after the first period. This reflects a tradeoff: higher labor in the tradable sector reduces consumption in the initial period but increases consumption in the following periods. In expected terms, utility under price control is lower initially but improves in the subsequent periods. The initial disadvantage of lower consumption is very high; it takes a long period for the cumulative utility to become positive. Eventually, price regulation yields higher overall utility compared to quantity regulation under the original parametrization of the model. Initially, quantity policy outperforms price policy due to lower labor share in the tradable sector. A higher labor share in the tradable sector accelerates technology accumulation, leading to higher TFP and higher overall output. Higher TFP, in return, reduces the labor demand of the tradable sector in the later periods. The spare labor can be redirected to produce more non-tradable goods, thereby improving household welfare.

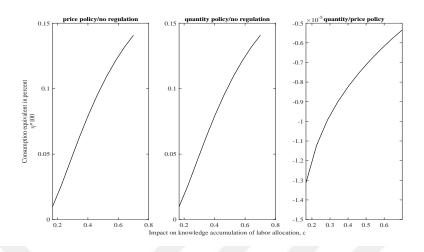
Which regulatory framework generates a higher welfare? In the previous part, we showed that under the BF parametrization of the model, optimal price policy generates higher welfare than optimal quantity policy in the long run. In the following parts, we extend our analysis to compare the welfare implications of price and quantity controls under alternative parametrizations of c and the initial TFP. Following the methodology utilized in BF, we measure welfare gains in terms of consumption. Welfare gain is expressed as the percentage of consumption that the representative household must receive to be indifferent between staying in the benchmark framework or moving to a new one. Welfare gain  $\eta$  is defined as,

$$\sum_{t=1}^{\infty} \beta^t log((1+\eta)c_t^{BF}) = \sum_{t=1}^{\infty} \beta^t log(c_t^{NF})$$
(4.60)

where the superscripts BF and NF denote allocations in the benchmark and new frameworks, respectively.

Welfare gains under alternative degrees of the impact on knowledge accumulation of labor allocation. The parameter c controls for the pace of knowledge accumulation. All other things being equal, higher c defines an economy with higher future output, a higher future price for the non-tradable goods, and closer proximity to the frontier (the last argument follows from eq. 4.4). Therefore, the magnitude of c





significantly influences the economy's transition and the regulatory policy response. Figure 4.9 shows welfare gains of optimal regulatory policies under various degrees of the impact on knowledge accumulation of labor allocation. The first two panels depict the welfare gains from optimal price and quantity policies, with the benchmark framework being the economy with no regulation. In the third panel, the benchmark framework is the economy with price regulation, and the new framework is the economy with quantity regulation. Higher *c* increases the welfare cost of neglecting the impact of labor allocation on TFP growth. Therefore, under both regulatory frameworks, the welfare gain with respect to the non-regulatory framework is an increasing function of *c*. The third panel reveals that under the original parametrization of the model and a range of values for *c*, where  $c \in [0.167, 0.641]$ , transitioning from price to quantity regulation worsens welfare.<sup>10</sup> However, welfare gain from price policy over quantity policy is decreasing in *c*. In the following, we will look at model dynamics under low and high *c* to better understand the welfare implications of price and quantity regulation.

<sup>&</sup>lt;sup>10</sup>The lower bound of the interval corresponds to the value given in the base scenario in Table 4.2, and we choose the upper bound so that the system's stability is ensured.

**Figure 4.10.** Quantity Policy: Expected Labor in the Tradable Sector and Expected Utility Along the Transition Path Under Alternative *c* 

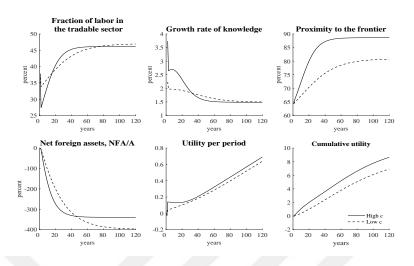
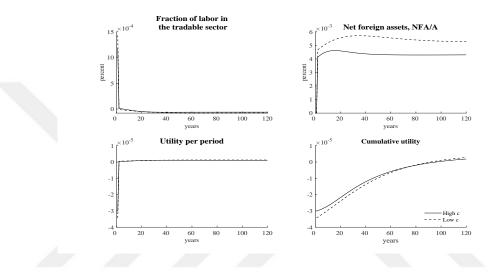


Figure (4.10) depicts the dynamics of the model under optimal quantity policy for low and high c. In the first period, with the regulation in place, the fraction of labor in the tradable sector increases. However, once the regulatory framework is out of the picture, the fraction of labor in the tradable sector declines, and this adjustment is notably much sharper under high c compared to low c. The divergence stems from the influence of the parameter c on the future stream of output and on the allocation of consumption between periods. In a scenario of high c, all others being equal, the growth rate of knowledge, output, and the price of non-tradables are higher in the later periods. Therefore, under high c, knowing that future output will be higher in the economy, households have a higher appetite to increase consumption in the early years. Furthermore, due to the expected higher future price of non-tradables, they want to consume more non-tradables earlier in the period. By substituting for  $A_{t+1}$ in the Euler equation for non-tradable goods, we can see that c directly affects the allocation of consumption between consecutive periods. Therefore, under high c, overall consumption at t = 1 is already high, and the consumption basket includes a higher proportion of non-tradables. Households tend to consume and borrow more

initially, and external debt increases sharply. Despite the lower share of labor in the tradable sector, the growth rate of knowledge is much higher initially under high *c*. As a result, households experience higher utility throughout all periods. Eventually, in the steady state, the economy is closer to the frontier and carries less debt.

**Figure 4.11.** Price vs. Quantity Policy: Expected Labor in the Tradable Sector and Expected Utility Along the Transition Path Under Alternative *c* 



The trends are similar under optimal price policy with small differences in magnitudes. In Figure 4.11, we compare the dynamics of the model under optimal price policy with that under optimal quantity policy for episodes of low and high *c*. Under higher *c*, the gap in expected labor allocated to the tradable sector is lower in the initial period. The response of optimal price regulation is related to the extent of variation in external debt following the realization of  $R_1$ . Under high *c*, the consumption of non-tradables is already high initially. Therefore, the response of labor and external debt to the variation in  $R_1$  is limited. Hence, the gap is smaller. Consider the response of the consumption path to  $R_1$ . In the case of a positive interest rate shock  $(R_1 > \mu_R)$ , households would want to consume less today and more tomorrow. Due to the sizable consumption of non-tradables today and the expected greater future output owing to high *c*, they can do that with a smaller adjustment in consumption today. In a scenario of a negative interest rate shock  $(R_1 < \mu_R)$ , households would want to consume more today. However, since they are already consuming a sizable amount, the shift of labor towards the production of non-tradable goods and the increase in debt would be limited. Under high *c*, the ex-post variation in external debt is low. If the variation is small, as demonstrated  $E(B_2(\tau^*))$  is closer to  $X^*$ . Consequently, the initial gap would be smaller, and so would the gap in later periods. The gap in the periods t > 1 favors price control. However, under high *c*, positive utility contributions in later periods do not fully compensate for the initial loss in utility. Consequently, the welfare gap ends up being lower under high *c*.

Welfare gains under alternative levels of initial TFP A higher initial TFP affects the output of tradables today and in future periods. Unlike the parameter c, initial TFP does not have a direct impact on the allocation of consumption across periods.

Figure 4.12. Price vs. Quantity Policy: Welfare Gains Under Alternative Initial TFP

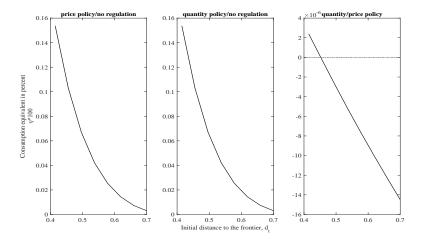


Figure 4.12 shows welfare gains of regulatory policies under alternative levels of initial TFP. In the first panel, the benchmark framework is the economy with no reg-

ulation, and the new framework is the one with price regulation. In the second panel, the benchmark framework is the economy with no regulation, and the new framework is the one with quantity regulation. In the third panel, benchmark framework is the economy with price regulation, and the new framework is the one with quantity regulation. We use the original parametrization except that we assume there is initially external debt ( $B_1 < 0$ ). This assumption ensures that households are always in debt and that the constraints on debt are binding for all possible values of initial TFP and interest rate in the first period. The horizontal axis represents the initial distance to the frontier, i.e.  $d_1$ . Welfare gains from having a regulatory framework compared to a non-regulatory framework decline with the initial TFP level. This occurs because, the response of  $A_{t+1}$  to  $L_t$  is decreasing in  $A_t$ . Taking derivative of  $A_{t+1}$  with respect to  $L_t$  and  $A_t$  we get,

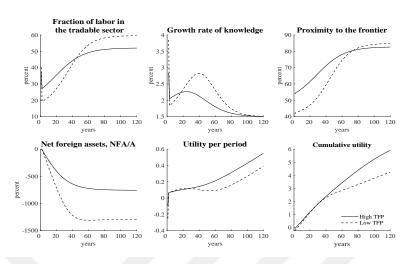
$$\frac{\partial^2 A_{t+1}}{\partial L_t^T \partial A_t} = c - 2d_t. \tag{4.61}$$

This derivative is less than zero as long as  $c/2 \le d_t$ , which holds true given the parametrization in the paper. The initial distance to the frontier,  $d_1$ , takes values in the range of [0.415, 0.659]. <sup>11</sup> In the initial panel, we show that for low levels of initial TFP, quantity policy improves welfare over price policy. However, this advantage of quantity policy declines as the initial level of TFP increases.

Figure 4.13 compares the transition to the steady state under optimal quantity policy for high and low initial TFP. The regulatory policy increases labor allocated to the tradable sector in the first period. In the second period, labor in the tradable sector declines under both scenarios but more significantly under low TFP. This occurs because, from the households' perspective, when TFP is low, it is less profitable to produce tradables; hence, less labor is allocated to the tradable sector, and the growth rate of knowledge is lower. Along the transition path, the labor share in the tradable

<sup>&</sup>lt;sup>11</sup>We choose boundaries that ensure the stability of the system.

**Figure 4.13.** Expected Model Dynamics Along the Transition Path Under Alternative Initializations of TFP



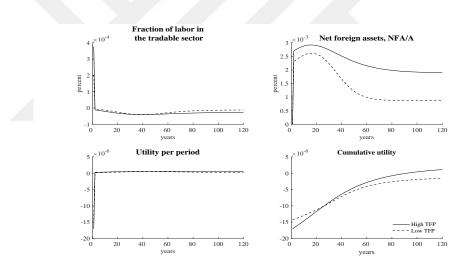
sector and the growth rate of knowledge increase much faster in the low TFP case. As labor growth in the tradable sector decelerates, the growth rate of knowledge declines, converging towards the world frontier. In the long run, although the economy is closer to the frontier, it is more indebted, and welfare is lower under the low TFP case.

Figure 4.14 shows the difference in the impact of regulatory policies during the transition of the economy, defined as price versus quantity regulation (gap in impact) under episodes of low and high initial TFP. Solid and dashed lines represent the gaps in impact under high and low initial TFP, respectively. At t = 1, the expected gap in labor increases relatively less under low TFP compared to the case of high TFP, resulting in a smaller drop in utility in the first period. This occurs because profits are lower in the tradable sector, and, therefore, consumption of non-tradable goods has a sizable weight in the consumption basket in the initial period. Therefore the response of  $B_2$  to  $R_1$  is small. Consider a scenario with a negative interest rate shock ( $R_1 < \mu_R$ ). Households would prefer to consume more today, but due to their already substantial consumption, the shift of labor towards the production of non-tradable goods and the

increase in debt would be limited.

Although the initial utility gap is higher under low TFP, the recovery in the aftermath is also more pronounced. Under an episode of low TFP, the initial gap in labor is smaller. Still, the fraction of labor in the tradable sector is so low initially that the growth rate of knowledge does not pick up for a considerable period during the transition. In other words, the initial gain in productivity caused by a higher share in the tradable sector does not bring about enough extra consumption in the later periods to compensate for the initial loss in utility. As a result, the social welfare gap remains positive in the long run, favoring quantity policy.

**Figure 4.14.** Price vs. Quantity policy: Expected Labor in the Tradable Sector and Expected Utility Along the Transition Path Under Alternative Initializations of TFP



**Volatility** The discussion around the figures 4.4 and 4.6 gives insight as to the volatility in the model. Accordingly, the volatility of consumption, production, and the real exchange rate is lower under the quantity measure. In this model, volatility does affect expected welfare through consumption and pollution. However, only the government faces uncertainty; agents decisions, on the other hand, are not affected by uncertainty. The welfare implication of higher volatility may be more significant

in a model where uncertainty directly impacts agents' decisions. For instance, consider a sunk cost model in which uncertainty worsens hysteresis losses. Baldwin and Krugman (1989) demonstrate that under the presence of sunk costs, volatility and persistence in the exchange rate could result in hysteresis in trade and investment. In the presence of sunk costs, hysteresis has significant implications for growth. The position of the firm determines its response to an exchange rate shock. Incumbent firms exit the market if the exchange rate falls below a threshold, and entrant firms enter the market if the exchange rate increases above a threshold. Between these thresholds lies the band of inaction, where incumbents remain in the market and potential entrants do not enter. Overvaluations in the exchange rate drive firms out of the market. Uncertainty widens the bands of inaction. Hence, strong fluctuations may lead to significant hysteresis losses (Belke et al. 2013).

#### 4.5. Conclusion

This chapter compares price and quantity-based capital inflow control measures in a small open economy that incorporates learning externalities in the tradable sector. We introduce uncertainty over the global interest rate to the Benigno and Fornaro (2014) small open economy model characterized by endogenous growth and a financial resource curse. In this model, households overlook the fact that productivity growth is a function of labor allocated to the tradable sector. Consequently, in the competitive equilibrium, labor allocated to the tradable sector is less than the socially optimal amount. There is information asymmetry such that the regulator sets the policy before observing the interest rate, agents, on the other hand, make decisions after observing the shock.

We show that there is less volatility in consumption, production, external debt and the exchange rate under quantity regulation. We study welfare under optimal price and optimal quantity policy. The optimal cap on external debt under quantity policy is higher than the expected debt implied by the optimal tax under price policy. This is because there is ex-post variation in external debt, and social welfare is concave and left skewed in external debt. Therefore, in terms of utility, quantity policy performs better than price policy in the short run. The higher the ex-post variation in external debt, the greater the relative advantage of quantity over price policy in the short term. The ranking of policies is influenced by the initial productivity level, where quantity (price) control performs better in terms of social welfare when the initial productivity level is low (high). The relative advantage of price over quantity policy declines with an increase in the pace of technology growth.

This chapter proposes a market-based basic quantity control scheme as an alternative policy tool to control capital inflows. Borrowing from the literature on price versus quantity controls, we propose a market-based regulatory framework where households require permits to borrow from the rest of the world.

## **CHAPTER V**

# CONCLUSION

This thesis aims to contribute to discussions about optimal regulation within the fields of environmental regulation and capital inflow controls. In Chapter II, acknowledging the interaction between the macro economy and the environment, we study the role of monetary policy when there are environmental concerns. Our study broadens a deterministic heterogeneous agent general equilibrium setup, where producers face cash-in-advance constraints in the labor market, with environment related components. These include a pollution externality, an abatement technology to partially contain it, and an environmental policy in the form of an emission tax. In Chapter III, we extend the setup introduced in Chapter II to explore the economy's response to productivity shocks under alternative environmental policies. We also investigate how the degree of nominal rigidity affects this response. The environmental policies considered encompass both price and quantity policies, represented by a tax on emissions and a partial market-based system respectively. The price of emissions remains fixed in the former, while in the latter, the price responds to the size of economic activity.

Shifting the focus to capital inflow controls in Chapter IV, we compare welfare implications of price and market-based quantity control measures in managing excessive capital inflows. We employ a small open economy model incorporating learning-bydoing externalities in the tradable sector.

In examining the interplay between monetary and environmental policies, our findings indicate that heterogeneity between agents in the economy in terms of pollution rates and consumption levels generates a role for monetary policy in enhancing social welfare and complementing regulatory efforts to address pollution. Similar to the outcomes observed in Annicchiarico and Di Dio (2015), the optimal monetary

policy tends to be more accommodating when the monetary authority considers the negative externality on the environment stemming from production. Addressing nominal rigidity implies higher output; therefore, given environmental concerns, monetary authority faces a trade-off between achieving a more efficient resource allocation with higher output and reducing pollution, which requires lower economic activity.

Our contribution to this literature emphasizes the distributional role of monetary policy. In our model, monetary policy has no direct influence on the abatement effort, but its impact on emissions is indirect, occurring via the change in labor allocation. This influence is limited compared to a regulatory policy. The impact of monetary policy on consumption is both direct and indirect. The direct impact operates through real wage adjustments and lump-sum money transfers, affecting consumption inequality. The indirect effect on consumption stems from the impact of monetary policy on optimal regulatory policy. Money growth shifts production away from the cash-constrained agents in the labor market. If these agents also happen to be the more pollutant type, this shift reduces overall emissions and allows room for more loose regulatory policy. Consequently, this leads to higher overall consumption and, subsequently, to elevated social welfare.

Incorporating uncertainty in our model setup enables an exploration of the impact of alternative regulatory policies on macro dynamics. Our second set of findings concerning the macro economy and the environment reveal that volatility is higher under a price policy compared to a quantity policy. The response of labor to a positive productivity shock is less pronounced under the quantity regulation. This occurs because the increase in output stimulates a price increase in the permits market, necessitating a larger allocation of labor to pollution control. This, in turn, raises the cost of production, weakening the response of labor to productivity shock. This result aligns with the findings in Fischer and Springborn (2011) and Annicchiarico and Di Dio (2015) that cap-and-trade mitigates volatility in main macroeconomic variables.

Furthermore, we demonstrate that as the cash constraint becomes tighter, with

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more labor expenses needing to be made in advance, variation of labor increases under both environmental policies, relatively more under a price policy with respect to a quantity policy. Aligned with the variation in labor, the variation of emissions under the price control increases as the cash constraint becomes more stringent. In Annicchiarico and Di Dio (2015) nominal rigidity is defined in the form of staggered price adjustment. They similarly conclude that the degree of price stickiness affects the ranking of alternative regulations.

The analysis presented in Chapters II and III might be enhanced by adopting the modeling strategy in Annicchiarico and Di Dio (2015) for representation of the abatement technology. This adjustment would allow us to benefit from the existent practice in calibrating parameters related to the environment. The present analysis in Chapter III has set the stage for a comprehensive comparison of optimal price and optimal quantity policies. Therefore, a plausible extension would involve conducting a fully-fledged welfare comparison of alternative environmental policies. This framework could further be employed to study the optimal response of monetary policy as we did in the previous chapter. The heterogeneity embedded in the model provides a ground for integrating the distributional effects of shocks and policies, which we show to have important implications for optimal policy design.

In Chapter IV, we compare the social welfare implications of the optimal price and the optimal quantity policy in the form of a cap-and-trade system for regulating capital inflows. To accomplish this, we extend the small open economy model in Benigno and Fornaro (2014), which incorporates learning externalities in the tradable sector. We introduce uncertainty over the global interest rate and a quantity-based alternative policy to regulate capital inflows. This chapter introduces a market-based quantity control scheme as an alternative policy tool to control capital inflows alongside taxes. Borrowing from the literature on price versus quantity controls, we propose a marketbased regulatory framework where households require permits to borrow from the rest of the world. Under the price policy, there exists a tax on external borrowing, while under the quantity policy, the government sets a cap on external borrowing and sells the rights to borrow in the spot market. Each permit grants the holder the right to borrow one unit of external debt. We study the impact of a downward shock in the global interest rate. Under the price policy, foreign debt changes in the realization of interest rate; under the quantity policy, foreign debt is fixed, and the realization of the interest rate affects the price of borrowing permits.

Our main findings in this part mirror the policy comparison under environmental control - under the quantity regulation, there is less volatility. Furthermore, in terms of utility, the quantity policy outperforms the price policy in the short run. This occurs because the expected external debt under the optimal price regulation is higher than the optimal cap on external debt set under quantity regulation. The uncertainty over external debt under the price regulation and the shape of the social welfare function contributes to this outcome. The greater the ex-post variation in external debt, the more pronounced the relative advantage of the quantity policy over the price policy in the short term. Moreover, we demonstrate that the ranking of policies is influenced by the initial productivity level, where the quantity (price) control performs better in terms of social welfare when the initial productivity level is low (high). The relative advantage of the price policy declines with an increase in the pace of technology growth.

The discussion of a market-based control mechanism represents a novel concept within the literature that studies the welfare implications of capital inflow controls. Traditionally, capital inflow controls are categorized as either price-based or quotabased administrative measures. The latter, involving quotas, are less favored due to concerns about susceptibility to rent-seeking behavior. Price-based controls come with their own caveat - when faced with uncertainty about the private sector's response, it might not be possible to reach desired quantities by setting prices.

In contrast, the cap-and-trade system, also termed a partial market-based approach in Mas-Colell et al. (1995), alleviates these concerns. It guarantees desired quantities without the risk of rent-seeking as rights to borrow is allocated through market mechanism. This type of measure is a viable candidate for capital inflow controls, provided there are enough agents to ensure that each is a price taker. The primary downside of the partial market-based approach compared to a price policy lies in its institutional complexity.

To date, the literature has demonstrated the cases where capital inflow controls improve welfare without distinguishing the type of control. We believe that much further research is required for the comparison of alternative policies. Concerning our research agenda, this chapter lays the groundwork for future studies that examine alternative policy options from a welfare perspective. For instance, the current model imposes control over borrowers, but an alternative design could focus on controlling foreign lenders. Furthermore, in this model, uncertainty does not directly affect the individual's decisions. The welfare implication of higher volatility may carry more significance in a model where uncertainty directly influences agents' decisions. Taking a step in this direction, an important avenue for future research could involve investigating the impact of alternative capital inflow control policies on the economy when sunk costs are involved in production activity. In the presence of sunk costs, growth potential is hindered by the hysteresis in trade and investment, and this effect is worsened by the extent of fluctuations in the exchange rate (Baldwin and Krugman 1989; Belke et al. 2013).



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## **APPENDIX**

#### **Proofs of Chapter II**

In this section, we provide the proofs of the propositions and the corollaries given in the main text. For ease of representation, input arguments of the functional forms are ignored. For instance,  $u_i(c_{it})$  appears as  $u_{it}$ .

**Proof of Proposition 1** In the following, we provide the solution of the stationary monetary competitive equilibrium. The Lagrangian of the model presented in equations (2.10)-(2.17) is given as:

$$\mathscr{L} = \sum_{t=0}^{\infty} \beta_{i}^{t} \left\{ \begin{array}{c} (u_{it} - B_{it}) + \\ \lambda_{1t} (\frac{M_{it} + (i-1)(\alpha_{i}/N_{i})M_{t}}{w_{t}} - L_{it}) + \\ \lambda_{2t} (p_{t}f_{it} + M_{i,t} + (\alpha_{i}/N_{i})M_{t} \\ -w_{t}L_{it} - \tau p_{t}E_{it} + T_{it} - M_{it+1}) \\ +\lambda_{3t} (M_{it+1} + (i-1)(\alpha_{i}/N_{i})M_{t+1}) \\ +\lambda_{4t} (M_{it+1} - w_{t+1}L_{it+1} + (\alpha_{i}/N_{i})M_{t+1}) \end{array} \right\}$$
(A.1)

$$c_{it} = (1 - n_i)f_{it} + \frac{M_{it} + (\alpha_i/N_i)M_t}{p_t} + \frac{T_i - \tau p_t E_{it}}{p_t} - \frac{w_t}{p_t} L_{it} - \frac{M_{it+1}}{p_t}$$
(A.2)

$$E_{it} = (1 - s_i(n_i))\gamma_i \tag{A.3}$$

$$E_t = \sum_i N_i E_{it} \tag{A.4}$$

#### First Order Conditions (FOCs) for Type 1 Agent

Given the assumptions stated in *Proposition 1*, only cash-in-advance for the labor market can be binding in the SMCE. Type 1 agent, as being the supplier of labor, is

not cash-constrained in the labor market, implying  $\lambda_{1t} = 0$ . Under the condition that  $\beta_1 < 1 + \alpha$ , type 1 agent has no motive to hold money in the SCME, hence  $M_{1t+1} = 0$ . All agents consume in the equilibrium, therefore  $\lambda_{2t} = 0$ . We further assume that when the labor and the goods markets open, agents have non-negative cash holdings, hence the associated constraints are not binding ( $\lambda_{3t} = \lambda_{4t} = 0$ ). Taking derivative of  $\mathscr{L}$  w.r.t.  $L_{1t}$  and imposing the equilibrium conditions we get:

$$\frac{\partial \mathscr{L}}{\partial L_{1t}} = 0 \tag{A.5}$$

$$u'_{1t} \left( \delta_1 f'_{1t} - \frac{w_t}{p_t} \right) = 0$$

$$f'_{1t} = \frac{1}{\delta_1} \frac{w_t}{p_t} \tag{A.6}$$

where  $\delta_1 = ((1 - n_1) - \tau_t (1 - s_1)\gamma_1)$  is the real revenue from unit of production net of emission tax.

Taking derivative of  $\mathscr{L}$  w.r.t.  $n_{1t}$  and imposing the equilibrium conditions we get:

$$\frac{\partial \mathscr{L}}{\partial n_{1t}} = 0$$

$$u'_{1t}(-1 + \tau_t s'_{1t} \gamma_1) f'_{1t} = 0$$

$$s'_{1t} = 1/(\tau_t \gamma_1)$$
(A.7)

Fraction of output reserved for reducing pollution  $n_{1t}$  is a function of  $\tau_t$  and  $\gamma_1$ .

Taking derivative of  $\mathscr{L}$  w.r.t.  $M_{1t+1}/p_t$  and imposing the equilibrium conditions we get:

$$\frac{\partial L}{\partial (M_{1t+1}/p_t)} \leq 0$$

$$-u'_{1t} + \beta_1 u'_{1t+1} \frac{p_t}{p_{t+1}} \leq 0.$$
(A.8)

In the SMCE,  $c_{1t} = c_{1t+1}$  and  $p_{t+1} = (1 + \alpha)p_t$ . Given the assumption that  $\beta_1 < (1 + \alpha)$ , then  $M_{t+1} = 0$ .

#### FOCs for Type 2 Agent

Given the assumptions stated in *Proposition 1*, type 2 agent demands labor in the SMCE and may be cash-constrained in the labor market  $(M_{2t+1} > 0, \lambda_{1t} \ge 0)$ . Taking derivative of  $\mathscr{L}$  w.r.t.  $L_{2t}$  and imposing the equilibrium conditions we get:

$$\frac{\partial \mathscr{L}}{\partial L_{2t}} = 0$$

$$u'_{2t} \left[ \delta_2 f'_{2t} - \frac{w_t}{p_t} \right] - \lambda_{1t} = 0$$
(A.9)

Taking derivative of  $\mathscr{L}$  w.r.t.  $n_2$  and imposing the equilibrium conditions we get:

$$s'_{2} = 1/(\tau \gamma_{2})$$
 (A.10)

Taking derivative of  $\mathscr{L}$  w.r.t.  $M_{2t+1}/p_t$  and imposing the equilibrium conditions we get:

$$\frac{\partial L}{\partial (M_{2t+1}/p_t)} = 0$$
 (A.11)  
$$-u'_{2t} + \beta_2 u'_{2t+1} \frac{p_t}{p_{t+1}} + \beta_2 \lambda_{1t+1} \frac{p_t}{w_{t+1}} = 0$$

Substituting in for  $\lambda_{1t}$  from FOC for  $L_{2t}$  we get:

$$-u_{2t}^{'} + \beta_2 u_{2t+1}^{'} \frac{p_t}{p_{t+1}} + \beta_2 u_{2t+1}^{'} (\delta_2 f_{2t+1}^{'} - \frac{w_{t+1}}{p_{t+1}}) \frac{p_t}{w_{t+1}} = 0$$
(A.12)

Imposing that in the SMCE,  $c_{2t} = c_{2t+1}$  and prices grow at the rate  $\alpha$ , FOC for  $M_{2t+1}/p_t$  becomes:

$$-1 + \beta_2 \frac{1}{1+\alpha} + \beta_2 (\delta_2 f'_{2t+1} - \frac{w_{t+1}}{p_{t+1}}) \frac{p_t}{w_{t+1}} = 0$$
 (A.13)

(A.14)

$$f_{2t}' = \frac{1+\alpha}{\delta_2 \beta_2} \frac{w_t}{p_t} \qquad (A.15)$$

Type 2 agents are constrained in the labor market. Therefore CIA constraint is binding for the type 2 agent:

$$w_t L_{2t} = M_{2t} + (\alpha_2 / N_2) M_t \tag{A.16}$$

#### Solution of the SMCE

Using (A.15), (A.9) and the labor market equilibrium condition we can solve for  $L_{1t}$ ,  $L_{2t}$ , and  $w_t/p_t$ :

$$\frac{\beta_2 \delta_2 f'_{2t}}{(1+\alpha)} = \frac{w_t}{p_t} = \delta_1 f'_{1t} (L_{1t} + \overline{L}_1)$$
(A.17)

Using the cash-in-advance constraint of the type 2 agent, we can solve for  $w_t$ :

$$w_{t} = \frac{M_{2t} + (\alpha_{2}/N_{2})M_{t}}{L_{2t}}$$

$$= \frac{(1 + \alpha_{2})M_{t}/N_{2}}{L_{2t}}$$
(A.18)

Second line follows from the fact that  $M_{1t} = 0$ . Using (A.9), we can solve for  $p_t$ :

$$p_t = \frac{(1+\alpha)}{\delta_2 \beta_2} \frac{w_t}{f_{2t}'} \tag{A.19}$$

Purchases by the first agent,  $q_{1t}$ , is derived from equation for money evolution imposing  $M_{1t+1} = 0$ :

$$p_{t}q_{1t} = (\alpha_{1}/N_{1})M_{t} - w_{t}L_{1t} - \tau p_{t}E_{1t} + T_{1t}$$

$$= (1 + \alpha)(M_{t}/N_{1}) - \tau p_{t}E_{1t} + T_{1t}$$

$$= -w_{t}L_{1t}\frac{1 + \alpha}{1 + \alpha_{2}} - \tau p_{t}E_{1t} + T_{1t}$$
(A.20)

Using goods market equilibrium condition and the government budget equation:

$$p_t q_{2t} = -w_t L_{2t} \frac{1+\alpha}{1+\alpha_2} + -\tau p_t E_{2t} + T_{2t}$$
(A.21)

Consumption of type 1 agent is given by:

$$c_{1t} = (1 - n_1)f_{1t} + q_{1t} \tag{A.22}$$

Substituting for purchases using equation (A.20) we get:

$$c_{1t} = (1 - n_1)f_{1t} - \frac{w_t}{p_t}L_{1t}\frac{1 + \alpha}{1 + \alpha_2} + \frac{T_{1t} - \tau p_t E_{1t}}{p_t}$$
(A.23)

Symmetrically, consumption of type 2 agent is given by

$$c_{2t} = (1 - n_2)f_{2t} - \frac{w_t}{p_t}L_{2t}\frac{1 + \alpha}{1 + \alpha_2} + \frac{T_{2t} - \tau p_t E_{2t}}{p_t}$$
(A.24)

Let  $M_0$  denote the initial distribution of money holdings.  $M_{20} = (1 - \mu)M_0$  and  $M_{10} = \mu M_0$ . Under the assumption that  $\beta_1 < 1 + \alpha$ , initial money holdings of the type 1 agent is also zero, i.e.  $\mu = 0$ . In the SMCE, money holdings for each individual grow at the same rate. Below, we show this feature of the equilibrium using the cash-in-advance constraint in the labor market:

$$w_{t}L_{2} = M_{2t} + (\alpha_{2}/N_{2})M_{t}$$

$$w_{t+1}L_{2} = M_{2t+1} + (\alpha_{2}/N_{2})M_{t+1}$$

$$w_{t}(1+\alpha)L_{2} = M_{2t+1} + (\alpha_{2}/N_{2})(1+\alpha)M_{t}$$

$$M_{2t}(1+\alpha) = M_{2t+1}$$
(A.25)

Since  $M_{1t} = 0$ ,  $M_{2t} = M_t / N_{2.}$ 

**Proof of Corollary 1.1** (*i*) Taking logarithm of both sides of the equality in (2.21) and differentiating w.r.t.  $\tau$  we get:

$$\frac{f_{1t}''}{f_{1t}'}\frac{\partial L_{1t}}{\partial \tau} + \frac{1}{\delta_1}\frac{\partial \delta_1}{\partial \tau} = -\frac{N_1 f_{2t}''}{N_2 f_{2t}'}\frac{\partial L_{1t}}{\partial \tau} + \frac{1}{\delta_2}\frac{\partial \delta_2}{\partial \tau}$$
(A.26)

$$\frac{\partial L_{1t}}{\partial \tau} = \frac{\frac{\gamma_1(1-s_1)}{\delta_1} - \frac{\gamma_2(1-s_2)}{\delta_2}}{\frac{f_{1t}''}{f_{1t}'} + \frac{N_1 f_{2t}''}{N_2 f_{2t}'}}$$
(A.27)

Given the assumptions about the production technology, the denominator of the above expression is negative. The ratio  $(1 - s_i)\gamma_i/\delta_i$  in the nominator represents emission per real revenue net of emission tax per one unit of output. Given that the denominator is negative,  $L_{1t}$  increases in response to a surge in tax if the nominator is negative. In turn, the nominator is negative if the type 2 agent is more pollutant. To prove this point, after substituting in for  $\delta_i$ , the nominator is negative if the following inequality holds:

$$\gamma_i(1-s_1)(1-n_2) - \gamma_2(1-s_2)(1-n_1) < 0 \tag{A.28}$$

Assuming an abatement technology as in equation (2.49), and further imposing that abatement technology is the same across agents ( $\varepsilon_1 = \varepsilon_2$ ), the above inequality simplifies to:

$$\frac{1}{1-\varepsilon}(n_2-n_1) < 0 \tag{A.29}$$

Note that, this inequality holds since  $n_i$  is increasing in  $\gamma_i$  and  $\varepsilon > 1$ , otherwise  $s_i$  would be greater than 1, which would unreasonably imply that any attempt to reduce pollution would effectively increase it.

Hence,  $\frac{\partial L_{1t}}{\partial \tau} < 0$  if type 1 agent is more pollutant;  $\frac{\partial L_{1t}}{\partial \tau} > 0$  if type 2 agent is more pollutant.

(ii) Assume that  $\overline{L}_2 = 0$  and  $f_{it} = (L_{it} + \overline{L}_i)^{\lambda_i}$  for i = 1, 2. Denote the real wage

 $\omega_t = w_t/p_t$ . The response of the real wage to the tax rate is given by:

$$\frac{\partial \omega_t}{\partial \tau} \frac{1}{\omega_t} = \frac{-(1-s_1)\gamma_1}{\delta_1} + \frac{\int_{1t}^{''}}{\int_1^{t}} \frac{\partial L_{1t}}{\partial \tau}$$
$$= \frac{-(1-s_1)\gamma_1}{\delta_1} + \frac{\int_{1t}^{''}}{\int_1^{t}} \frac{L_{1t} + \overline{L}_1}{L_{1t} + \overline{L}_1} \frac{\partial L_{1t}}{\partial \tau}$$
$$= \frac{-(1-s_1(n_1))\gamma_1}{\delta_1} + (\lambda_1 - 1) \frac{1}{L_{1t} + \overline{L}_1} \frac{\partial L_{1t}}{\partial \tau}$$

Substituting for  $\partial L_{1t}/\partial \tau$  using equation (A.26), we get:

$$= \frac{-(1-s_{1})\gamma_{1}}{\delta_{1}} + (\lambda_{1}-1)\frac{1}{L_{1t}+\bar{L}_{1}}\frac{\frac{(1-s_{1})\gamma_{1}}{\delta_{1}} - \frac{(1-s_{2})\gamma_{2}}{\delta_{2}}}{\frac{f_{1t}''}{f_{1t}'} + \frac{N_{1}f_{2t}''}{N_{2}f_{2t}'}}$$
(A.30)  
$$= \frac{-(1-s_{1})\gamma_{1}}{\delta_{1}} + \frac{\frac{(1-s_{1})\gamma_{1}}{\delta_{1}} - \frac{(1-s_{2})\gamma_{2}}{\delta_{2}}}{1 - \frac{(L_{1t}+\bar{L}_{1})(1-\lambda_{2})}{L_{1t}(1-\lambda_{1})}}$$
  
$$= \frac{\frac{(1-s_{1})\gamma_{1}}{\delta_{1}}\frac{(L_{1t}+\bar{L}_{1})(1-\lambda_{2})}{L_{1t}(1-\lambda_{1})} - \frac{(1-s_{2})\gamma_{2}}{\delta_{2}}}{1 - \frac{(L_{1t}+\bar{L}_{1})(1-\lambda_{2})}{L_{1t}(1-\lambda_{1})}} < 0$$

Notice that since  $L_{1t} < 0$ , the denominator is positive and the nominator is negative. Hence the whole expression is less than zero, implying that higher tax reduces real wage.

**Proof of Corollary (1.2)** Taking the logarithm of both sides of the equality in (2.21) and differentiating w.r.t.  $\alpha$  we get:

$$\frac{f_{1t}''}{f_{1t}'} \frac{\partial L_{1t}}{\partial \alpha} = -\frac{N_1 f_{2t}''}{N_2 f_{2t}'} \frac{\partial L_{1t}}{\partial \alpha} - \frac{1}{1+\alpha}$$
(A.31)
$$\frac{\partial L_{1t}}{\partial \alpha} = \frac{-\frac{1}{1+\alpha}}{\frac{f_{1t}''(L_{1t}+\overline{L}_1)}{f_{1t}'(L_{1t}+\overline{L}_1)} + \frac{N_1 f_{2t}''(L_{2t}+\overline{L}_2)}{N_2 f_{2t}'(L_{2t}+\overline{L}_2)}} > 0$$

$$\frac{\partial L_{2t}}{\partial \alpha} = -\frac{N_1}{N_2} \frac{\partial L_{1t}}{\partial \alpha} < 0$$
(A.32)

$$\frac{\partial(w_t/p_t)}{\partial\alpha} = \delta_1 f_{1t}''(L_{1t} + \overline{L}_1) \frac{\partial L_{1t}}{\partial\alpha} < 0$$
(A.33)

The proof is complete.

**Proof of Proposition 2** In the following, we provide a solution to the social planners' problem. Taking derivative of the social welfare function w.r.t.  $\psi$  we get:

$$u'_1 = u'_2$$
 (A.34)

Taking derivative w.r.t.  $n_{it}$  we get:

$$-\left(\psi u_{1}^{'}+(1-\psi)u_{2}^{'}\right)N_{i}f_{it}+\left(N_{1}B_{1}^{'}+N_{2}B_{2}^{'}\right)\left(N_{i}s_{it}^{'}\gamma_{i}f_{it}\right) = 0 \quad (A.35)$$

 $B'\gamma_i s'_{it} = u'$  (A.36)

where  $u' = \psi u'_1 + (1 - \psi)u'_2$  and  $B' = N_1B'_1 + N_2B'_2$ . Taking derivative w.r.t.  $L_{1t}$  we get:

$$\left(\psi u_{1}^{'} + (1-\psi)u_{2}^{'}\right)\frac{\partial c_{t}}{\partial L_{1t}} - \left(N_{1}B_{1}^{'} + N_{2}B_{2}^{'}\right)\frac{\partial E_{t}}{\partial L_{1t}} = 0$$

(A.37)  $u'\frac{\partial c_{t}}{\partial L_{1t}} - B'\frac{\partial E_{t}}{\partial L_{1t}} = 0$ (A.38)  $\left((1-n_{1})f_{1t}' - (1-n_{2})f_{2t}'\right)N_{1}u' - \left((1-s_{1t})\gamma_{1}f_{1t}' - (1-s_{2t})\gamma_{2}f_{2t}'\right)N_{1}B' = 0$ (A.39)

**Proof of Corollary 2.1** Parts (*i*) and (*ii*) directly follow from equation (2.40). For part (*iii*) I need to show that  $(1 - n_i) - (1 - s_i)\gamma_i B'/u'$  is decreasing in  $\gamma_i$ . Substituting for  $\gamma_i B'/u'$  using (2.39), this expression becomes,  $(1 - n_i) - (1 - s_i)/s'_i$ . It's derivative

w.r.t  $\gamma_i$  is:

$$\frac{(1-s_i)s_i''}{(s_i')^2}\frac{\partial n_i}{\partial \gamma_i}$$
(A.40)

Taking derivative of (2.39) w.r.t.  $\gamma_i$  we get  $\partial n_i / \partial \gamma_i = -s'_i / s''_i \gamma_i$ . Substituting for  $\partial n_i / \partial \gamma_i$  in the expression above, we get

$$\frac{(1-s_i)s_i''}{(s_i')^2}\frac{\partial n_i}{\partial \gamma_i} = -\frac{(1-s_i)}{s_i'\gamma_i} < 0.$$
(A.41)

The proof is complete.

**Proof of Proposition 3** (i) Here, we present the derivations for the optimal monetary and regulatory policy under competitive equilibrium where redistribution of taxes in a socially optimal way is possible. First order conditions for optimal regulatory and monetary policies are given by:

$$a'\frac{\partial C_t}{\partial \tau} - B'\frac{\partial E_t}{\partial \tau} = 0$$
 (A.42)

$$u'\frac{\partial C_t}{\partial \alpha} - B'\frac{\partial E_t}{\partial \alpha} = 0 \tag{A.43}$$

Derivatives of total consumption and total pollution w.r.t.  $\tau$  and  $\alpha$  are given by:

$$\frac{\partial C_{t}}{\partial \tau} = N_{1} \left( -\frac{\partial n_{1}}{\partial \tau} f_{1t} + (1 - n_{1}) f_{1t}^{'} \frac{\partial L_{1t}}{\partial \tau} \right)$$

$$+ N_{2} \left( -\frac{\partial n_{2}}{\partial \tau} f_{2t} + (1 - n_{2}) f_{2t}^{'} \frac{\partial L_{2t}}{\partial \tau} \right)$$
(A.44)

$$\frac{\partial E_t}{\partial \tau} = N_1 \gamma_1 \left( -\frac{\partial n_1}{\partial \tau} s'_1 f_{1t} + (1-s_1) f'_1 \frac{\partial L_{1t}}{\partial \tau} \right)$$
(A.45)

$$+N_2\gamma_2\left(-\frac{\partial n_2}{\partial \tau}s_2'f_{2t}+(1-s_2)f_{2t}'\frac{\partial L_{2t}}{\partial \tau}\right)$$
(A.46)

$$\frac{\partial C_t}{\partial \alpha} = N_1 \left( (1 - n_1) f_{1t}' \frac{\partial L_{1t}}{\partial \alpha} \right) + N_2 \left( (1 - n_2) f_{2t}' \frac{\partial L_{2t}}{\partial \alpha} \right)$$
(A.47)

$$\frac{\partial E_t}{\partial \alpha} = N_1 \gamma_1 \left( (1 - s_1) f_{1t}' \frac{\partial L_{1t}}{\partial \alpha} \right) + N_2 \gamma_2 \left( (1 - s_2) f_{2t}' \frac{\partial L_{2t}}{\partial \alpha} \right)$$
(A.48)

Combining (A.43), (A.47) and (A.48) we get:

$$\left((1-n_1)f_{1t}' - (1-n_2)f_{2t}'\right)u' = \left((1-s_1)\gamma_1f_{1t}' - (1-s_2)\gamma_2f_{2t}'\right)B'$$
(A.49)

Combining (A.42), (A.44) and (A.45) we get:

$$N_1 \left( -\frac{\partial n_1}{\partial \tau} f_{1t} \right) \left( u' - B' s_1' \gamma_1 \right) \tag{A.50}$$

$$+N_2\left(-\frac{\partial n_2}{\partial \tau}f_{2t}\right)\left(u'-B's_2'\gamma_2\right)+\tag{A.51}$$

$$N_{1} \frac{\partial L_{1t}}{\partial \tau} \left\{ \left( (1 - n_{1}) f_{1t}^{'} - (1 - n_{2}) f_{2t}^{'} \right) u^{'} - \left( (1 - s_{1}) \gamma_{1} f_{1t}^{'} - (1 - s_{2}) \gamma_{2} f_{2t}^{'} \right) B^{'} \right\}$$
(A.52)

Substituting from (2.23) and (A.49) in the above expression we get:

$$\left(u^{'}-B^{'}\frac{1}{\tau}\right)\left(-\frac{\partial n_{1}}{\partial \tau}N_{1}f_{1t}-\frac{\partial n_{2}}{\partial \tau}N_{2}f_{2t}\right)=0.$$

This equation is satisfied when:

$$\tau = \frac{B'}{u'} \tag{A.53}$$

Substituting (A.53) back in (A.49) for  $\frac{B'}{u'}$  we get:

$$\delta_1 f_{1t}' = \delta_2 f_{2t}'$$

This condition complies with the equilibrium condition (A.49) of the SMCE when:

$$\beta_2 = (1 + \alpha)$$

The proof is complete.

(*ii*) Here, we present the derivations for the optimal monetary and regulatory policy under competitive equilibrium where redistribution of taxes in a socially optimal way is not possible.

$$c_{it} = (1 - n_i)f_i - \omega_t L_{it} \frac{(1 + \alpha)}{(1 + \alpha_2)}$$
 (A.54)

$$E = \sum_{i} N_i (1 - s_i) \gamma_i f_{it} \tag{A.55}$$

Maximize SW w.r.t.  $\alpha$ :

$$\frac{\partial SW}{\partial \alpha} = N_1 u'_{1t} \frac{\partial c_{1t}}{\partial \alpha} + N_2 u'_{2t} \frac{\partial c_{2t}}{\partial \alpha} - B' \frac{\partial E_t}{\partial \alpha}$$
(A.56)

where  $B' = N_1 B'_1 + N_2 B'_2$ .

$$\frac{\partial SW}{\partial \alpha} = N_1 u_1' ((1-n_1) f_{1t}' \frac{\partial L_{1t}}{\partial \alpha} - \omega_t \frac{(1+\alpha)}{(1+\alpha_2)} \frac{\partial L_{1t}}{\partial \alpha} 
-L_{1t} \left( \frac{\partial \omega_t}{\partial \alpha} \frac{(1+\alpha)}{(1+\alpha_2)} + \frac{\omega_t}{(1+\alpha_2)} \right) ) 
+N_2 u_{2t}' \left( (1-n_2) f_{2t}' \frac{\partial L_{2t}}{\partial \alpha} - \omega_t \frac{(1+\alpha)}{(1+\alpha_2)} \frac{\partial L_{2t}}{\partial \alpha} 
-L_{2t} \left( \frac{\partial \omega_t}{\partial \alpha} \frac{(1+\alpha)}{(1+\alpha_2)} + \frac{\omega_t}{(1+\alpha_2)} \right) \right) 
-B' \left( N_1 (1-s_1) \gamma_1 f_{1t}' \frac{\partial L_{1t}}{\partial \alpha} + N_2 (1-s_2) \gamma_2 f_{2t}' \frac{\partial L_{2t}}{\partial \alpha} \right) 
= 0$$
(A.57)

Collecting similar terms, we get:

$$\frac{\partial SW}{\partial \alpha} = \left(u_1'(1-n_1)f_1' - u_2'(1-n_2)f_2'\right)\frac{\partial L_{1t}}{\partial \alpha} \\
-B'\left((1-s_1)\gamma_1f_1' - (1-s_2)\gamma_2f_2'\right)\frac{\partial L_{1t}}{\partial \alpha} \\
-\omega_t\frac{(1+\alpha)}{(1+\alpha_2)}\frac{\partial L_{1t}}{\partial \alpha}\left(u_1' - u_2'\right) \\
-L_{1t}\left(\frac{\partial \omega_t}{\partial \alpha}\frac{(1+\alpha)}{(1+\alpha_2)} + \frac{\omega_t}{(1+\alpha_2)}\right)\left(u_1' - u_2'\right) \\
= 0 \quad (A.59)$$

We substitute in for  $\omega_t$  and  $\frac{\partial \omega_t}{\partial \alpha}$ . We divide the whole expression by  $f'_{1t}$  and use the condition  $\frac{\partial L_{1t}}{\partial \alpha}$  and use  $\frac{f'_{2t}}{f'_{1t}} = \frac{\delta_1(1+\alpha)}{\delta_2\beta_2}$ :

$$\begin{aligned} \frac{\partial SW}{\partial \alpha} &= \left( u_1'(1-n_1) - u_2'(1-n_2) \frac{\delta_1(1+\alpha)}{\delta_2 \beta_2} \right) \\ &- B' \left( (1-s_1)\gamma_1 - (1-s_2)\gamma_2 \frac{\delta_1(1+\alpha)}{\delta_2 \beta_2} \right) \\ &- \delta_1 \frac{(1+\alpha)}{(1+\alpha_2)} \left( u_1' - u_2' \right) \\ &- L_{1t} \left( \delta_1 \frac{f_1''}{f_1'} \frac{(1+\alpha)}{(1+\alpha_2)} \right) \left( u_1' - u_2' \right) \\ &- L_{1t} \left( \frac{\delta_1}{(1+\alpha_2)} \right) \left( u_1' - u_2' \right) / \frac{\partial L_{1t}}{\partial \alpha} \\ &= 0 \end{aligned}$$

We substitute in for  $\frac{\partial L_{1t}}{\partial \alpha}$  using equation (A.31):

$$\frac{\partial SW}{\partial \alpha} = \left( u'_{1t}(1-n_1) - u'_{2t}(1-n_2) \frac{\delta_1(1+\alpha)}{\delta_2 \beta_2} \right) 
-B' \left( (1-s_1)\gamma_1 - (1-s_2)\gamma_2 \frac{\delta_1(1+\alpha)}{\delta_2 \beta_2} \right) 
-\delta_1 \frac{(1+\alpha)}{(1+\alpha_2)} \left( u'_1 - u'_2 \right) 
-L_{1t} \left( \delta_1 \frac{f''_{1t}}{f'_{1t}} \frac{(1+\alpha)}{(1+\alpha_2)} \right) \left( u'_{1t} - u'_{2t} \right) 
+L_{1t} \left( \frac{\delta_1(1+\alpha)}{(1+\alpha_2)} \right) \left( u'_{1t} - u'_{2t} \right) \left( \frac{f''_{1t}}{f'_{1t}} + \frac{N_1 f''_{2t}}{N_2 f'_{2t}} \right) 
= 0$$
(A.60)

Simplifying we get:

$$\frac{(1+\alpha)}{\beta_2} = \frac{\frac{\left(u'_{1t}(1-n_1)-B'(1-s_1)\gamma_1\right)}{\delta_1}}{\left(\frac{u'_{2t}(1-n_2)-B'(1-s_2)\gamma_2}{\delta_2}\right) + \left(\frac{\beta_2}{(1+\alpha_2)}\right)\left(u'_{1t}-u'_{2t}\right)\left(1+\frac{f''_{2t}L_{2t}}{f'_{2t}}\right)}$$
(A.61)

Maximizing SW w.r.t.  $\tau$ , we get:

$$\frac{\partial SW}{\partial \tau} = N_1 u_1' \frac{\partial C_{1t}}{\partial \tau} + N_2 u_2' \frac{\partial C_{2t}}{\partial \tau} - B' \frac{\partial E_t}{\partial \tau}$$
(A.62)

$$\frac{\partial SW}{\partial \tau} = N_1 u_1' \left( -\frac{\partial n_1}{\partial \tau} f_{1t} + (1-n_1) f_{1t}' \frac{\partial L_{1t}}{\partial \tau} - \frac{(1+\alpha)}{(1+\alpha_2)} \left( \frac{\partial \omega_t}{\partial \tau} L_{1t} + \omega_t \frac{\partial L_{1t}}{\partial \tau} \right) \right) \\
+ N_2 u_{2t}' \left( -\frac{\partial n_2}{\partial \tau} f_{2t} + (1-n_2) f_{2t}' \frac{\partial L_{2t}}{\partial \tau} - \frac{(1+\alpha)}{(1+\alpha_2)} \left( \frac{\partial \omega_t}{\partial \tau} L_{2t} + \omega_t \frac{\partial L_{2t}}{\partial \tau} \right) \right) \\
- B' \left( N_1 \left( -s_1' \gamma_1 f_{1t} \frac{\partial n_1}{\partial \tau} + (1-s_1) \gamma_1 f_{1t}' \frac{\partial L_{1t}}{\partial \tau} \right) + N_2 \left( -s_2' \gamma_2 f_{2t} \frac{\partial n_2}{\partial \tau} + (1-s_2) \gamma_2 f_{2t}' \frac{\partial L_{2t}}{\partial \tau} \right) \right) \\
= 0 \qquad (A.63)$$

Collecting terms and using  $s'_i \gamma_i = 1/\tau$ , we get:

$$\frac{\partial SW}{\partial \tau} = -\frac{\partial n_1}{\partial \tau} N_1 f_{1t} \left( u'_{1t} - \frac{B'}{\tau} \right) - \frac{\partial n_2}{\partial \tau} N_2 f_{2t} \left( u'_{2t} - \frac{B'}{\tau} \right) \quad (A.64)$$

$$+ N_1 \frac{\partial L_{1t}}{\partial \tau} \left\{ \left( (1 - n_1) u'_{1t} f'_{1t} - B'(1 - s_1) \gamma_1 f'_{1t} \right) - \left( (1 - n_2) u'_{2t} f'_{2t} - B'(1 - s_2) \gamma_2 f'_{2t} \right) \right\} \quad (A.65)$$

$$- \frac{(1 + \alpha)}{(1 + \alpha_2)} \frac{\partial \omega_t}{\partial \tau} L_{1t} N_1 \left( u'_{1t} - u'_{2t} \right) - \frac{(1 + \alpha)}{(1 + \alpha_2)} \omega_t \frac{\partial L_{1t}}{\partial \tau} N_1 \left( u'_{1t} - u'_{2t} \right)$$

$$= 0$$

We substitute in for  $\frac{\partial \omega_t}{\partial \tau}$  and  $\omega$ , and divide the whole expression by  $f'_{1t}$ , and  $L_{1t}N_1$ :

$$\frac{\partial SW}{\partial \tau} = -\frac{\partial n_1}{\partial \tau} \frac{f_{1t}}{f'_{1t}} \frac{1t}{L_{1t}} \left( u'_{1t} - \frac{B'}{\tau} \right) 
- \frac{\partial n_2}{\partial \tau} \frac{N_2}{N_1} \frac{f_{2t}}{f'_{1t}} \frac{1t}{L_{1t}} \left( u'_{2t} - \frac{B'}{\tau} \right) 
+ \frac{\partial L_{1t}}{\partial \tau} \frac{1}{L_{1t}} \left\{ \left( (1 - n_1)u'_{1t} - B'(1 - s_1)\gamma_1 \right) 
- \left( (1 - n_2)u'_{2t} \frac{f_{2t}}{f'_{1t}} - B'(1 - s_2)\gamma_2 \frac{f_{2t}}{f'_{1t}} \right) \right\} 
+ \frac{(1 + \alpha)}{(1 + \alpha_2)} \left( u'_{1t} - u'_{2t} \right) ((1 - s_1)\gamma_1) 
- \frac{(1 + \alpha)}{(1 + \alpha_2)} \left( u'_{1t} - u'_{2t} \right) \delta_1 \frac{f''_{1t}}{f'_{1t}} \frac{\partial L_{1t}}{\partial \tau} 
- \frac{(1 + \alpha)}{(1 + \alpha_2)} \delta_1 \frac{\partial L_{1t}}{\partial \tau} \frac{1}{L_{1t}} \left( u'_{1t} - u'_{2t} \right)$$
(A.66)

Substituting in for the expression in curly brackets from  $\frac{\partial SW}{\partial \alpha} = 0$  (eq. A.59), using labor market equilibrium condition and simplifying we get:

$$\frac{\partial SW}{\partial \tau} = -\frac{\partial n_1}{\partial \tau} \left( \frac{f_{1t}}{f_{1t}'} \frac{1t}{L_{1t}} \right) \left( u_{1t}' - \frac{B'}{\tau} \right) 
+ \frac{\partial n_2}{\partial \tau} \left( \frac{f_{2t}}{f_{2t}'} \frac{1}{L_{2t}} \right) \frac{\delta_1 (1+\alpha)}{\beta_2 \delta_2} \left( u_{2t}' - \frac{B'}{\tau} \right) 
+ \left( \frac{\partial L_{1t}}{\partial \tau} / \frac{\partial L_{1t}}{\partial \alpha} \right) \left( \frac{\delta_1}{(1+\alpha_2)} \right) \left( u_{1t}' - u_{2t}' \right) 
+ \frac{(1+\alpha)}{(1+\alpha_2)} \left( u_{1t}' - u_{2t}' \right) (1-s_1) \gamma_1 
= 0$$
(A.67)

Substitute in for  $\left(\frac{\partial L_{1t}}{\partial \tau} / \frac{\partial L_{1t}}{\partial \alpha}\right)$  and simplifying we get:

$$\frac{\partial SW}{\partial \tau} = -\frac{\partial n_1}{\partial \tau} \left( \frac{f_{1t}}{f'_{1t}} \frac{1}{L_{1t}} \right) \left( u'_{1t} - \frac{B'}{\tau} \right)$$
(A.68)

$$+\frac{\partial n_2}{\partial \tau} \left(\frac{f_{2t}}{f'_{2t}} \frac{1}{L_{2t}}\right) \frac{\delta_1(1+\alpha)}{\beta_2 \delta_2} \left(u'_{2t} - \frac{B'}{\tau}\right)$$
(A.69)

$$+\frac{(1-s_{2})\gamma_{2}}{\delta_{2}}\left(\frac{\delta_{1}(1+\alpha)}{(1+\alpha_{2})}\right)\left(u_{1t}^{'}-u_{2t}^{'}\right)$$
(A.70)

$$= 0$$
 (A.71)

Substituting in for  $f'_{1t}$  and  $L_{1t}$ , and simplifying we get:

$$\frac{\partial SW}{\partial \tau} = \left(\frac{f_{2t}}{f'_{2t}}\frac{1}{L_{2t}}\right)\frac{(1+\alpha)}{\beta_2}\left(\frac{N_1}{N_2}\frac{\partial n_1}{\partial \tau}\frac{f_{1t}}{f_{2t}}u'_{1t} + \frac{\partial n_2}{\partial \tau}u'_{2t}\right) 
- \frac{B'}{\tau}\left(\frac{f_{2t}}{f'_{2t}}\frac{1}{L_{2t}}\right)\frac{(1+\alpha)}{\beta_2}\left(\frac{N_1}{N_2}\frac{\partial n_1}{\partial \tau}\frac{f_{1t}}{f_{2t}} + \frac{\partial n_2}{\partial \tau}\right) 
+ (1-s_2)\gamma_2\left(\frac{(1+\alpha)}{(1+\alpha_2)}\right)\left(u'_{1t} - u'_{2t}\right) 
= 0$$
(A.72)

$$\frac{B'}{\tau} = \frac{\left(\frac{N_1}{N_2}\frac{\partial n_{1t}}{\partial \tau}\frac{f_1}{f_2}u'_1 + \frac{\partial n_{2t}}{\partial \tau}u'_2\right)}{\left(\frac{N_1}{N_2}\frac{\partial n_{1t}}{\partial \tau}\frac{f_1}{f_2} + \frac{\partial n_{2t}}{\partial \tau}\right)} + \frac{(1-s_2)\gamma_2\left(\frac{(1+\alpha)}{(1+\alpha_2)}\right)\left(u'_1 - u'_2\right)}{\left(\frac{f_2}{f_2'}\frac{1}{L_{2t}}\right)\frac{(1+\alpha)}{\beta_2}\left(\frac{N_1}{N_2}\frac{\partial n_{1t}}{\partial \tau}\frac{f_1}{f_2} + \frac{\partial n_{2t}}{\partial \tau}\right)}$$
(A.73)

## Loglinearization Methodology

We log linearize the system using the following approximation:

$$x_t \approx \overline{x}e^{\widehat{x}_t}$$
, where  $\widehat{x}_t = \frac{x_t - \overline{x}}{\overline{x}}$ . (A.74)

We replace the variables  $c_{it}$ ,  $f_{it}$ ,  $\frac{w_t}{p_t}$ ,  $L_{it}$  in levels according to (A.74), then we compute linear approximation taking derivatives with respect to  $\hat{x}_t$ . Random variables  $z_{it}$ 

are not transformed. We take derivatives with respect to the variable itself. In the transformed system  $z_{it}$  are in levels, other variables are in percentage deviation from the the steady state. First order approximation is given by:

$$f(\widehat{\mathbf{x}}_t) \approx f(\overline{x}) + D(\overline{x}) \times \widehat{\mathbf{x}}_t, \tag{A.75}$$

where  $\hat{\mathbf{x}}_t$  is the vector of variables in the form of percentage change (except for  $z_{it}$ ) and *D* is the vector of partial derivatives.  $\bar{x}$  represents the steady state.

## **Quadratic Approximation to the Social Welfare Function**

In the numerical exercise presented in section xx, we use the following equations:

$$\frac{\partial SW}{\partial L_2 \partial L_2} = \sum_i N_i u_i'' \left(\frac{\partial c_i}{\partial L_2}\right)^2 + N_i u_i' \left(\frac{\partial^2 c_i^k}{\partial L_2 \partial L_2}\right) - \frac{\partial^2 B}{\partial E \partial E} \left(\frac{\partial E}{\partial L_2}\right)^2 - \frac{\partial B}{\partial E} \left(\frac{\partial^2 E}{\partial L_2 \partial L_2}\right)$$
(A.76)

$$\frac{\partial SW}{\partial L_2 \partial z_2} = N_2 u_2'' \frac{\partial c_2}{\partial z_2} \frac{\partial c_2}{\partial L_2} + N_2 u_2' \frac{\partial^2 c_2}{\partial L_2 \partial z_2} - \frac{\partial^2 B}{\partial E \partial E} \frac{\partial E}{\partial z_2} \frac{\partial E}{\partial L_2} - \frac{\partial B}{\partial E} \left(\frac{\partial^2 E}{\partial L_2 \partial z_2}\right)$$
(A.77)

$$\frac{\partial SW}{\partial z_2 \partial z_2} = N_2 u_2'' \left(\frac{\partial c_2}{\partial z_2}\right)^2 + N_2 u_2' \frac{\partial^2 c_2}{\partial z_2 \partial z_2} - \frac{\partial^2 B}{\partial E \partial E} \left(\frac{\partial E}{\partial z_2}\right)^2 - \frac{\partial B}{\partial E} \frac{\partial^2 E}{\partial z_2 \partial z_2}$$
(A.78)

Taking derivative of  $c_1$  w.r.t.  $L_2$  we get:

$$\frac{\partial c_1}{\partial L_2} = \frac{N_2}{N_1} \left( f_1'' L_1 \frac{1 + \alpha_2 + \theta \alpha_1}{1 + \alpha_2} + f_1' L_1 \frac{\theta \alpha_1}{1 + \alpha_2} \right)$$
(A.79)

$$\frac{\partial^2 c_1}{\partial L_2 \partial L_2} = -\left(\frac{N_2}{N_1}\right)^2 \left(f_1''' L_1 \frac{1 + \alpha_2 + \theta \alpha_1}{1 + \alpha_2} + f_1'' \frac{1 + \alpha_2 + 2\theta \alpha_1}{1 + \alpha_2}\right)$$
(A.80)

Taking derivative of  $c_2$  w.r.t.  $L_2$  and  $z_2$  we get:

$$\frac{\partial c_2}{\partial L_2} = \frac{\partial \delta_2}{\partial L_2} f_2 + \delta_2 f_2' + \left(\frac{N_2}{N_1} f_1'' L_2 - f_1'\right) \left(\frac{1 + \alpha 2 + \theta \alpha_1}{1 + \alpha_2}\right)$$
(A.81)

$$\frac{\partial c_2}{\partial z_2} = \frac{\partial \delta_2}{\partial z_2} f_2 + \delta_2 f_2 \tag{A.82}$$

$$\frac{\partial^2 c_2}{\partial L_2 \partial L_2} = \frac{\partial^2 \delta_2}{\partial L_2 \partial L_2} f_2 + 2 \frac{\partial \delta_2}{\partial L_2} f_2' + \delta_2 f_2'' + \frac{N_1}{N_2} \left( 2f'' - \frac{N_1}{N_2} f_1''' L_2 \right) \left( \frac{1 + \alpha 2 + \theta \alpha_1}{1 + \alpha_2} \right)$$
(A.83)

$$\frac{\partial^2 c_2}{\partial L_2 \partial z_2} = \frac{\partial \delta_2}{\partial L_2} f_2 + \delta_2 f_2' + \frac{\partial^2 \delta_2}{\partial L_2 \partial z_2} f_2 + \frac{\partial \delta_2}{\partial z_2} f_2'$$
(A.84)

$$\frac{\partial^2 c_2}{\partial z_2 \partial z_2} = f_2 \left( \frac{\partial \delta_2}{\partial z_2 \partial z_2} + 2 \frac{\partial \delta_2}{\partial z_2} + \delta_2 \right)$$
(A.85)

Taking derivatives of  $\delta_2$  w.r.t.  $L_2$  and  $z_2$  we get:

$$\frac{\partial \delta_2}{\partial L_2} = \frac{\partial \delta_2}{\partial (p^e/p)} \frac{\partial (p^e/p)}{\partial L_2} = -\frac{(1-s_2)\gamma_2\lambda_2\varepsilon}{\varepsilon - 1} \frac{(p^e/p)}{L_2}$$
(A.86)

where, using equation xx,  $\frac{\partial(p^e/p)}{\partial L_2} = \lambda_2 \frac{\varepsilon}{\varepsilon - 1} \frac{(p^e/p)}{L_2}$ , and  $\frac{\partial(p^e/p)}{\partial z_2} = \frac{\varepsilon}{\varepsilon - 1} (p^e/p)$ .

$$\frac{\partial^2 \delta_2}{\partial L_2 \partial L_2} = \frac{\gamma_2 \lambda_2 \varepsilon}{\varepsilon - 1} \frac{(p^e/p)}{L_2} \left( \frac{\partial s_2}{\partial L_2} - \frac{(1 - s_2)(1 + \lambda_2)\varepsilon}{\varepsilon - 1} \right)$$
(A.87)

$$\frac{\partial \delta_2}{\partial z_2} = \frac{\partial \delta_2}{\partial (p^e/p)} \frac{\partial (p^e/p)}{\partial z_2} = -\frac{(1-s_2)\gamma_2\varepsilon}{\varepsilon - 1} (p^e/p)$$
(A.88)

$$\frac{\partial^2 \delta_2}{\partial z_2 \partial z_2} = \frac{\gamma_2 \varepsilon(p^e/p)}{\varepsilon - 1} \left( \frac{\partial s_2}{\partial z_2} - \frac{(1 - s_2)\varepsilon}{\varepsilon - 1} \right)$$
(A.89)

$$\frac{\partial^2 \delta_2}{\partial z_2 \partial L_2} = \gamma_2 \frac{\varepsilon}{\varepsilon - 1} \frac{p^e}{p} \left( \frac{\partial s_2}{\partial L_2} - \frac{(1 - s_2)\lambda_2 \varepsilon}{\varepsilon - 1} \frac{1}{L_2} \right)$$
(A.90)

Under the price regulation, price on emissions is fixed, therefore, terms involving partial derivative of  $\delta_2$  are zero. Taking derivatives of  $E_2$  w.r.t.  $L_2$  and  $z_2$  we get:

$$\frac{\partial E_2}{\partial L_2} = N_2 \gamma_2 \left( -\frac{\partial s_2}{\partial L_2} f_2 + (1 - s_2) f_2' \right) \tag{A.91}$$

$$\frac{\partial E_2}{\partial z_2} = N_2 \gamma_2 f_2 \left( -\frac{\partial s_2}{\partial z_2} + (1 - s_2) \right)$$
(A.92)

$$\frac{\partial^2 E_2}{\partial L_2 \partial L_2} = N_2 \gamma_2 \left( -\frac{\partial^2 s_2}{\partial L_2 \partial L_2} f_2 - 2 \frac{\partial s_2}{\partial L_2} f_2' + (1 - s_2) f_2'' \right)$$
(A.93)

$$\frac{\partial^2 E_2}{\partial L_2 \partial z_2} = \frac{\partial E_2}{\partial L_2} - N_2 \gamma_2 \left( \frac{\partial^2 s_2}{\partial L_2 \partial z_2} f_2 + \frac{\partial s_2}{\partial L_2} f_2' \right)$$
(A.94)

$$\frac{\partial^2 E_2}{\partial z_2 \partial z_2} = \frac{\partial E_2}{\partial z_2} - N_2 \gamma_2 f_2 \left( \frac{\partial^2 s_2}{\partial z_2 \partial z_2} + \frac{\partial s_2}{\partial z_2} \right)$$
(A.95)

Taking derivatives of  $s_2$  w.r.t.  $L_2$  and  $z_2$  we get:

$$\frac{\partial s_2}{\partial L_2} = \frac{\kappa \lambda_2}{\varepsilon - 1} \frac{n^{1-\varepsilon}}{L_2}$$
(A.96)

$$\frac{\partial s_2}{\partial z_2} = \frac{\kappa}{\varepsilon - 1} n_2^{1 - \varepsilon} \tag{A.97}$$

$$\frac{\partial^2 s_2}{\partial z_2 \partial z_2} = -\frac{\kappa}{\varepsilon - 1} n_2^{1 - \varepsilon}$$
(A.98)

$$\frac{\partial^2 s_2}{\partial L_2 \partial L_2} = -\frac{(1+\lambda_2)}{L_2} \frac{\partial s_2}{\partial L_2}$$
(A.99)

$$\frac{\partial^2 s_2}{\partial L_2 \partial z_2} = -\frac{\partial s_2}{\partial L_2} \tag{A.100}$$

The impact of  $z_2$  and  $L_2$  on  $s_2$  is through prices in the permits market. We use equations for  $s_2$ ,  $n_2$ , and  $(p^e/p)$  for the derivations above.

# **TÜRKÇE ÖZET**

Bu tez, dışsallıkları düzenlemenin makro ekonomik sonuçlarını araştıran üç makaleden oluşmaktadır. İlk iki makale, makroekonomik politikalar ile çevre politikaları arasındaki etkileşimi genel denge modelleri çerçevesinde inceleyen yazına katkıda bulunmayı amaçlamaktadır. Üçüncü makale, dışsallıkları düzenlemede öne çıkan fiyat ve miktar kontrolleri kıyaslamasını sermaye girişi kontrol önlemleri alanına uygulamaktadır.

Dışsallıklar, özellikle çevresel kaygılar özelinde, uzun süredir iktisatçıların araştırma gündeminin bir parçası olmuştur. Ekonomideki aktörlerin davranışlarından doğan dışsal etkiler rekabetçi piyasa altında Pareto optimal olmayan sonuçlara yol açar. Bu etkiler, bir kişinin faydası başka bir kişinin yaptığı seçimlerden doğrudan etkilendiğinde ortaya çıkar. Pareto optimal dengeyi yeniden sağlamaya yönelik çözümler ekonomi yazınında ayrıntılı bir şekilde ele alınmıştır (Mas-Colell vd., 1995). Bu çözümler, vergi, kota ve dışsallık hakkının alınıp satıldığı piyasa bazlı önlemleri kapsamaktadır. Belirsizliğin yokluğunda, vergiler ve kotalar Pareto etkin dengenin olusmasını sağlar. Dahası, eğer dışsallıklar iyi tanımlanırsa ve uygulanabilir mülkiyet hakları oluşturulabilirse, dışsallık izinlerine yönelik rekabetçi piyasalar da Pareto optimal dengeyi sağlama kapasitesine sahiptir. Günümüzde çevre kirliliği kontrolü için kullanılan piyasa bazlı mekanizmalar (cap-and-trade systems) da, kısmi piyasa bazlı kontrol adıyla iktisat yazınında ortaya konmuştur. Bu yaklaşım altında devlet toplam dışsallık seviyesini belirler ve her biri bir birim dışsallık üretme hakkını temsil eden ticarete konu dışsallık izinlerini dağıtır. Bu izinlerin ticareti yoluyla piyasa etkin bir dengeye ulaşır.

Dışsallıkların kontrolü uzun bir süredir araştırma konusu olurken, makro politikalar ile kontrol politikaları, özellikle çevre politikaları, arasındaki etkileşimi konu alan araştırmalar nispeten yenidir. Zaman içinde çevre kontrolünün öneminin giderek daha fazla anlaşılmasına paralel olarak ekonominin çevre üzerindeki etkisine ilişkin farkındalık da artmıştır. Bir taraftan, iş çevrimleri çevre kirliliği ve çevre koruma politikalarının tasarımını etkilemektedir. Diğer taraftan, çevre politikalarının tasarımı, ekonominin şoklara verdiği tepkiyi etkilemektedir. Bu etkileşimler, makroekonomik ve çevre politikalarının doğru tasarımına ilişkin soruları gündeme getirmektedir. Makro politikalar doğrudan kirliliği kontrol etmeyi amaçlamayabilir. Bununla birlikte, çevre politikalarının ekonominin dinamiklerini etkilediği durumda, istikrarı korumayı amaçlayan makro politikaların bu etkileri dikkate alacak şekilde düzenlenmesi ihtiyacı ortaya çıkabilecektir.

İktisat yazınında, ekonomi, ekonomi politikası, çevre ve çevre politikaları arasın-

daki etkileşimleri anlamak amacıyla çevresel unsurlar ile makroekonomiyi Dinamik Stokastik Genel Denge (DSGE) modelleri çerçevesinde birleştiren bir alan bulunmaktadır. Örneğin, iş çevrimlerinin çevre politikasına etkisine odaklanan Heutel (2012) ve Ramezani vd. (2020) sabit çevre vergisi yerine döngüyle birlikte hareket eden değişken çevre vergisi önermektedir.

Fischer ve Springborn (2011) çalışması verimlilik şokları altında çevre politikası tercihlerinin (vergi, piyasa bazlı miktar kısıtı, yoğunluk hedefi) ekonomi üzerindeki dinamik etkilerini karşılaştırmaktadır. Benzer şekilde, Anniccharico ve Di Dio (2015) ekonominin alternatif çevresel düzenlemeler kapsamında nominal ve reel şoklara tepkisini araştırmaktadır. Ayrıca farklı çevre politikası rejimleri altında enflasyona optimal politika tepkisini de incelemektedir.

Dissou ve Karnizova(2016), piyasa mekanizması bazlı miktar kısıtı ve vergi düzenlemerininin ekonomiye etkisini sektöre özgü verimlilik şoklarının varlığında değerlendirmektedir. Başka bir çalışmada Annicchiarico ve Di Dio (2017), çevre kirliliğinin ekonomiye olumsuz etkilediği Yeni Keynesyen bir modelde para politikasının çevre politikasıyla etkileşimini araştırmaktadır. Bulgular, para politikasının çevre politikasını etkilediğini ve çevresel kaygıların da optimal para politikasının tasarımını etkilediğini vurgulamaktadır. Nominal katılıklara odaklanarak üretimin artırılmasının daha fazla oranda çevre kirliliğine yol açabileceği düşünüldüğünde, katı enflasyon hedeflemesi varsayımı artık optimal sayılmamaktadır. Bu alandaki araştırmalar, genel olarak para ve çevre politikası arasındaki etkileşimi ortaya koyarak çevresel kaygıların optimal para politikasının tasarımını etkilediğini vurgulamaktadır.

Para politikası ile çevrenin etkileşimi büyüme modelleri kullanılarak da incelenmektedir. Faria (1998) ve Faria vd. (2023), çevreyi hem fayda fonksiyonuna hem de üretim sürecine katkıda bulunan yenilenebilir bir varlık (bir sermaye türü) olarak ele alarak para politikasının çevre üzerindeki etkisini bir büyüme modeli kapsamında ele almaktadır. Bulguları para politikasının çevresel etkileri olabileceğine işaret etmektedir.

Bu tezin ilk iki makalesi, ekonomi ve çevre politikalarının etkileşimini inceleyen bu literatüre katkıda bulunmayı amaçlamaktadır. II. Bölüm'de nakit avans modeli çerçevesinde para ve çevre politikalarının etkileşimi çalışılmaktadır. Bu kapsamda, üretim bağlantılı çevre kirliliğinin yer aldığı ve iktisadi faaliyeti yürüten aktörlerin peşin ödeme kısıtına tabi olduğu deterministik bir genel denge modeli kurulmuştur. Başçı ve Sağlam (2005) heterojen aktör nakit avans modeline Kelly (2005) modelindeki çevresel unsurlar dahil edilmiştir. Bu model çerçevesinde, para politikası ile çevre kirliliğini hedef alan vergi politikasının genel denge ve refah etkileri incelenmiş, söz konusu iki politikanın etkileşimi araştırılmıştır.

Çalıştığımız modelde para politikası sosyal refahı, üretim ve tüketim kanallarından dolaylı ve doğrudan etkilemektedir. Para politikasının çevre kirliliği ile mücadele üzerinde doğrudan etkisi yoktur. Bununla birlikte kaynakların, verimlilik ve karbon emisyon oranları açısından farklılık gösteren aktörler arasındaki dağılımını etkilemek suretiyle, çevre kirliliğini dolaylı olarak etkilemektedir. Para politikasının tüketim üzerindeki doğrudan etkisi reel ücretler ve para transferi yoluyla ortaya çıkmaktadır. Tüketim üzerindeki dolaylı etkisi ise para politikası ve çevre politikasının etkileşimi üzerinden ortaya çıkmaktadır. Nakit avans kısıtına tabi olan üreticilerin aynı zamanda daha yüksek oranda çevreyi kirleten kesim olması durumunda, para arzındaki hızlanma, üretim dağılımının karbon emisyonu daha yüksek olan üretici grubu aleyhinde değişmesine ve çevre kirliliğinin azalmasına neden olmaktadır. Çevre kirliğindeki azalma ise çevre vergisinin azaltılabilmesi için bir alan oluşturmakta, çevre vergisindeki azalma da tüketimin artmasına neden olmaktadır. Özetle, bu çalışma, ekonominin karbon emisyon oranları ve verimlilik boyutunda farklılaşan ajanlardan oluştuğu bir yapıda, çevresel kaygıların para politikasının tasarımını etkileyebileceğine işaret etmektedir.

Anniccharico ve Di Dio(2015) çalışmasının çıkarımlarına benzer şekilde, optimal para politikası, üretimin çevreyi kirleten boyutunu dikkate aldığında daha gevşek olma eğilimindedir. Para politikasının nominal katılıkların etkisinin yok edilmesine odaklanması daha yüksek üretim, dolayısıyla daha fazla çevre kirliliği anlamına gelmektedir. Bu nedenle, çevresel kaygılar göz önüne alındığında, parasal otorite, daha yüksek üretimle daha verimli kaynak tahsisi sağlamak ile daha düşük ekonomik aktivite gerektiren kirliliği azaltmak arasında bir tercihle karşı karşıyadır.

İktisat yazınına göre, belirsizliğin olduğu durumlarda, herhangi bir dışsallığın kontrol edilmesinde fiyat veya miktar düzenlemelerinin kullanılması refahı farklı şekilde etkilemektedir (Weitzman, 1974). Tezin ikinci makalesinde (III. Bölüm), tezin II. Bölümü'nde kullanılan deterministik genel denge modeline, verimlilik şokları eklenmiştir. Bu eklemeler sonucunda, üretim bağlantılı çevre kirliliğinin yer aldığı ve üreticilerin işgücü piyasasında kısmi peşin ödeme kısıtına tabi olduğu rassal bir genel denge modeli kurulmuştur. Modele belirsizliği dahil etmek, alternatif çevre politikalarının makro dinamikler üzerindeki etkisinin araştırılmasına olanak sağlamaktadır.

Tezin bu kısmında ilgi duyulan temel sorular, sistemin farklı çevre politikaları altında verimlilik şoklarına nasıl tepki verdiği ve kısmi peşin ödeme kısıtı şeklinde modele dahil edilen nominal katılığın derecesinin bu tepkiyi nasıl etkilediğidir. Karşılaştırılan çevre düzenlemeleri vergi ve piyasa temelli miktar kontrolünden oluşmaktadır. Piyasa temelli miktar kontrolü altında kamu otoritesi toplam karbon emisyon üst sınırını belirler ve her biri bir birim dışsallık üretme hakkını temsil eden ticarete konu emisyon izinlerini üreticilerin talep eden konumunda olduğu piyasaya arz eder. Bu durumda, çevreyi kirletmenin bedeli piyasanın dengeye ulaştığı noktada belirlenir. Dolayısıyla, vergi düzenlemesi kapsamında çevreyi kirletme karşılığı ödenen bedel sabitken, piyasa temelli miktar düzenlemesi kapsamında kirletme bedeli iktisadi faaliyet ile aynı yönde hareket etmektedir.

Temel bulgularımız iki yönlüdür. Birincisi, iktisat yazını ile uyumlu olarak, fiyat düzenlemesi altında makro değişkenlerdeki oynaklık, miktar düzenlemesine kıyasla daha yüksektir. İkincisi, nominal katılığın derecesi arttıkça oynaklık her iki düzenlemede de artmakta, ancak artış fiyat düzenlemesinde nispeten daha yüksek oranda olmaktadır. İşgücünün miktar düzenlemesi altında daha sınırlı tepki vermesinin nedeni, üretimdeki artışın izin piyasasında fiyat artışını tetiklemesi ve bunun da üretim maliyetini yükseltmesidir.

Tezin son makalesinin yer aldığı IV. Bölüm'de odak noktası ilk iki bölümden farklı olarak sermaye girişi kontrolleridir. Bu kısımda uluslararası sermaye hareketlerinin kısıtlanmasına ihtiyaç duyulan bir çerçeve ele alınmakta, bu kapsamda fiyat ve miktar tipi düzenlemelerin sosyal refaha etkisi karşılaştırılmaktadır. Bu amaçla Benigno ve Fornaro (2014) modeline küresel likidite şoku ve alternatif sermaye kontrolü düzenlemeleri dahil edilmiştir.

Ticarete konu olan ve olmayan iki sektörün yer aldığı dışa açık küçük bir ekonomi modeli baz alınmaktadır. Bu ekonomide, hane halkları dünyanın geri kalanından borçlanarak tüketimini zamanlar arası dengeleme imkanına sahiptir. Modelde hane halkının refah açısından optimal düzeyin üzerinde borçlanmasına neden olan bir dışsallık bulunmaktadır. Ticarete konu olan sektörde teknoloji gelişimi, yaparak öğrenme (learning-by-doing) esasına dayanmaktadır. Hane halklarının borçlanma ve tüketim kararlarını alırken bunu dikkate almaması yakın dönemde hane halkının refah açısından optimal düzeyin üzerinde tüketmesine ve borçlanmasına neden olmaktadır. Bu nedenle modelde dışarıdan borçlanmanın düzenlenmesi gerekmektedir. Modelde aynı zamanda bilgi asimetrisi bulunmaktadır. Kamu otoritesi düzenleme yaptığı aşamada yurt dışından borçlanmanın maliyeti hakkında bilgi sahibi değildir. Diğer taraftan hane halkları borçlanma maliyetini gördükten sonra karar almaktadır. Bu çerçevede, beklenti üzerinden hareket etmek zorunda kalan otorite tarafından uygulanan optimal fiyat ve miktar politikalarının toplam refaha etkisi farklı olmaktadır.

Söz konusu çerçeve kullanılarak, birinde fiyat diğerinde miktar düzenlemesi olan iki ayrı model kurgulanmıştır. Fiyat düzenlemesi kapsamında, kamu otoritesinin rolü dış borçlanmaya vergi koymaktır. Miktar düzenlemesi altında ise kamu otoritesi toplam dış borçlanmaya bir üst sınır getirmekte ve her biri bir birim borçlanma hakkını temsil eden borçlanma izinlerini spot piyasada arz etmektedir. Hane halkı bu izinleri kamunun tek tedarikçi olduğu spot piyasadan satın alabilmektedir. Her iki düzenlemede de düzenleyicinin politikayı önceden belirlemesi nedeniyle bilgi asimetrisi söz konusudur.

Üçüncü makale iktisat yazınına iki şekilde katkıda bulunmayı amaçlamaktadır: İlk olarak, fiyat ve miktar kontrolleri üzerine literatürden esinlenerek, sermaye girişleri için fiyat (vergi) ve piyasa mekanizmasını temel alan miktar tipi düzenlemenin refah etkilerini karşılaştırmaktadır. İkinci olarak, sermaye girişleri için piyasa mekanizmasına dayalı kontrol kavramını gündeme getirmektedir.

Sayısal analize göre, miktar politikası altında kısa vadede beklenen fayda, fiyat politikasına kıyasla daha yüksek olmaktadır. Bu durum fiyat politikası altında dış borcun değişkenlik göstermesi ve bu oynaklığın beklenen sosyal refahı azaltıcı etkisinden kaynaklanmaktadır. Kamu otoritesi miktar politikası kullandığında, fiyat politikasının ima ettiği beklenen borç seviyesinin üzerinde bir borç tavanı belirleyebilmektedir. Bu da kısa vadede tüketimin miktar politikası altında fiyat politikasına kıyasla daha yüksek olmasına neden olmaktadır. Dolayısıyla borcun volatilitesi arttıkça miktar politikasının kısa vadedeki göreli avantajı da artmaktadır.

Politikaların sosyal refaha etkisi bağlamında karşılaştırılmasında başlangıç teknoloji seviyesinin önemli olduğu gözlenmiştir. Başlangıç teknoloji seviyesi düşük olduğunda miktar politikası fiyat politikasına kıyasla daha iyi sonuç vermektedir. Teknolojik büyümenin hızı arttıkça fiyat politikasının miktar politikası üzerindeki göreli avantajı azalmaktadır.



# CURRICULUM VITAE Hatice Burcu GÜRCİHAN

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M. Res. in Economics, Barcelona Graduate School of Economics, Spain, 2008.M. Res. in Economics, Middle East Technical University, 2004.BSc in Economics, Middle East Technical University, 2001.

## **Publications**

## **Refereed Journal Articles**

"Real wages and the business cycle in Turkey", with Altan Aldan, Acta Oeconomica, 72(1), 105-121.

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*"Risk analysis of the government domestic debt stock in Turkey: cost-at-risk approach"*, Middle East Technical University Economics Department Master Thesis.

## **Other Professional Experience**

**Co-Editor** : Working Papers and Research Notes in Economic Series, Central Bank of Turkey, As of 2024/1.